Simulation of D.C. Machines Transient Behaviors: Teaching and Research.

O.I. Okoro, Ph.D., C.U. Ogbuka, B.Eng., and M.U. Agu, Ph.D.

Department of Electrical Engineering, University of Nigeria, Nsukka, Enugu State.

*E-mail: ucogbuka@yahoo.com
oiokoro@hotmail.com
drmarcelagu@yahoo.co.uk

ABSTRACT

Teaching electrical machines transient behaviors can be very difficult and stressful. This is because the resulting systems of differential equations are usually non-linear and difficult to solve analytically to the comprehension of the students. In order to make the topic easier, simplifying assumptions are used which do not necessarily represent actual machine behaviors. This paper, therefore, sets out to model various types of D.C. machines using the block-approach method of SIMULINK®. The analysis carried-out and the results obtained can be very useful in the study of the D.C. motor dynamics, controllability, observability, and stability.

(Keywords: D.C. machines, direct current, transient, stability, SIMULINK, modeling)

INTRODUCTION

The applications of D.C. machines have remained vital in industrial processes notwithstanding some competitive advantages of AC machines.

Large D.C. motors are used in machine tools, printing presses, conveyor fans, pumps, hoists, cranes, paper mills, textile mills, and so on. Small D.C. motors (in fractional power rating) are used in control devices such as tacho-generators for speed sensing and servomotors for positioning and tracking [1].

The increasing complexity of industrial processes demand greater flexibility from electrical machines in terms of special characteristics and speed control [2]. Their torque/speed characteristics can be varied over a wide range, while the machine maintains its efficiency without sacrificing its speed as in the case of induction motors [3]. D.C. motors have always been used in applications in where good controllability is demanded. The greatest advantage of D.C. machines over other types of machines is the ease of their speed control [4].

This paper presents the transient performances of the various classes of D.C. machines (in motor mode) using SIMULINK®. The electrical and mechanical equations of each class are developed in state space form from which the SIMULINK® models are built. The startup responses are generated for further analysis.

SIMULINK MODELS OF D.C. MACHINES

Separately Excited D.C Machines

As the name implies, the field winding in a separately excited D.C. machine is connected to its own D.C. source, independent of the armature circuit. The armature current during starting can be much greater than its rated value. This can be dangerous to the commutator, which should not be exposed to current greater than twice the rated current. Therefore separate and shunt excited D.C. motor are better connected to the source via voltage regulators such as choppers or rectifiers [4]. A circuit model of a Separately Excited D.C. motor is shown in Figure 1 below.

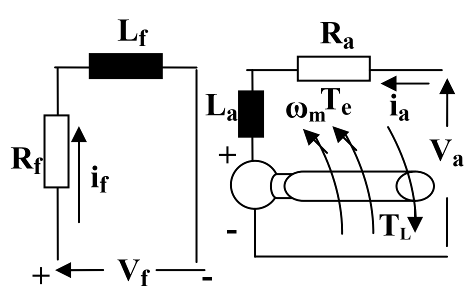

Figure 1: A Separately Excited D.C. Motor.

The dynamic equations describing the behavior of the separately excited D.C. machine are:

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + K_b i_f \omega_m \qquad \textbf{(1)}$$

$$V_f = R_f i_f + L_f \frac{di_f}{dt} \qquad \textbf{(2)}$$

The equations are related to the operation of the machine with a rigidly connected inertia load with moment of inertia (J) and the friction losses in the load are represented by a viscous friction coefficient (B) [5] as shown in Figure 1. Then the torque developed by the machine (in motor mode) is represented as:

$$T_e = T_l + B\omega_m + J\frac{d\omega_m}{dt} \qquad \textbf{(3)}$$

Where ω_m is the tool angular velocity (mechanical rad/sec) and T_l is the load torque.

Generally, the machine torque is related to the armature and field current as:

$$T_e = K_a * \Phi * i_a \qquad \textbf{(4)}$$

For a separately excited D.C. machine, the flux is established by a separate field excitation.

$$\Phi = \frac{N_f * i_f}{S} = K_f i_f$$

Where, N_f is the number of turns in the field winding and S is the reluctance of the structure. Therefore, for a separately excited D.C. motor,

$$T_e = K_b * i_a * i_f = T_l + B\omega_m + J\frac{d\omega_m}{dt} \qquad \textbf{(5)}$$

State Space representation allows an nth-order continuous system to be represented by a set of n simultaneous first-order differential equations [6]. Therefore, the electrical and mechanical models suitable for dynamic simulation are realized from Equation 1, 2, 3, and 5 as:

$$\begin{bmatrix} \dot{i}_a \\ \\ \dot{i}_f \end{bmatrix} = \begin{bmatrix} \dfrac{-R_a}{L_a} & \dfrac{-K_b\omega_m}{L_a} \\ 0 & \dfrac{-R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_a \\ \\ i_f \end{bmatrix} + \begin{bmatrix} \dfrac{V_a}{L_a} \\ \dfrac{V_f}{L_f} \end{bmatrix} \qquad \textbf{(6)}$$

$$T_e = K_b * i_a * i_f \qquad \textbf{(7)}$$

$$\dot{\omega}_m = \frac{1}{J}(T_e - B\omega_m - T_l) \qquad \textbf{(8)}$$

The SIMULINK® model for the Separately Excited D.C. Motor and the submodels of each of the subsystems are shown in Figure 2 below:

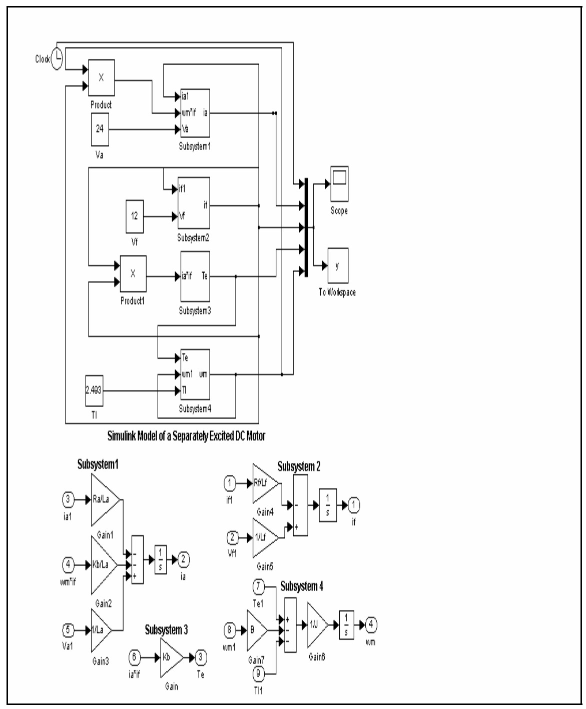

Figure 2: Simulink and Subsystem Models of Separately Excited D.C. Motor.

Shunt D.C. Machines

The field coils of a shunt D.C. machine are connected in parallel to the armature coils. Therefore, the transient in the armature circuit is simultaneous with the transient in the field circuit [5]. A circuit model of a Shunt D.C. Motor is shown in Figure 3 below:

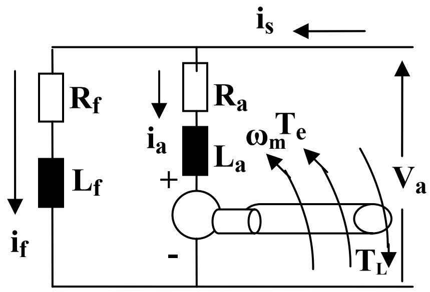

Figure 3: A Shunt D.C. Motor.

The dynamic equations for shunt D.C. motor are:

$$V_a = i_a R_a + L_a \frac{di_a}{dt} + K_b i_f \omega_m \qquad \textbf{(9)}$$

$$Va = i_f R_f + L_f \frac{di_f}{dt} \qquad \textbf{(10)}$$

Owing to the simultaneous excitation of both the field and the armature winding,

$$T_e = K_b * i_a * i_f = T_l + B\omega_m + J\frac{d\omega_m}{dt} \qquad \textbf{(11)}$$

The dynamic equations suitable for simulation are:

$$\begin{bmatrix} \dot{i}_a \\ \\ \dot{i}_f \end{bmatrix} = \begin{bmatrix} \frac{-R_a}{L_a} & \frac{-K_b\omega_m}{L_a} \\ \\ 0 & \frac{-R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_a \\ \\ i_f \end{bmatrix} + \begin{bmatrix} \frac{V_a}{L_a} \\ \\ \frac{V_a}{L_f} \end{bmatrix} \qquad \textbf{(12)}$$

$$T_e = K_b * i_a * i_f \qquad \textbf{(13)}$$

$$\dot{\omega}_m = \frac{1}{J}(T_e - B\omega_m - T_l) \qquad \textbf{(14)}$$

The SIMULINK® model for the Shunt D.C. Motor and the sub-models of each of the subsystems are shown in Figure 4 below.

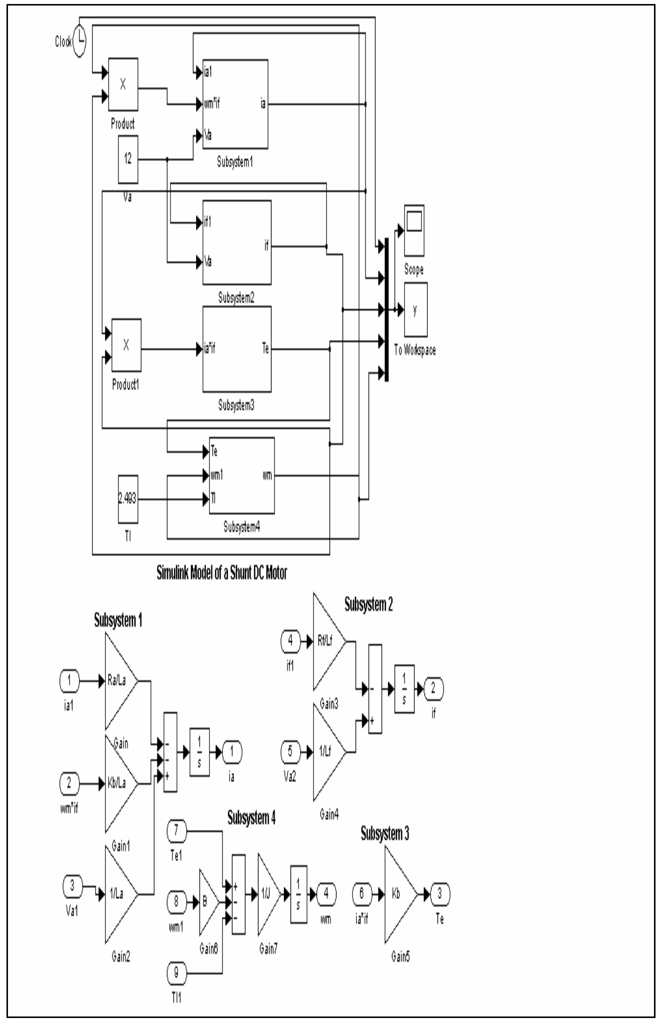

Figure 4: SIMULINK and Subsystem models of Shunt D.C. Motor.

Series Wound D.C. Machines

The armature current of a series D.C. motor is at the same time its field current. This means that the speed of series D.C. motors increase as the load decreases.

At no load (zero armature current) the speed will theoretically become infinity [4]. A circuit model of a Series D.C. Motor is shown in Figure 5 below.

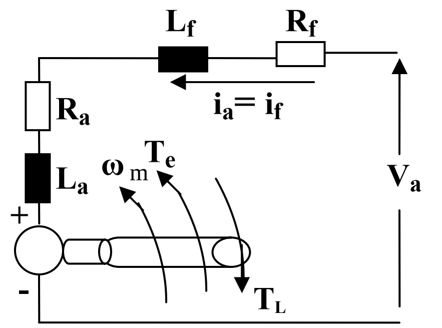

Figure 5: A Series D.C. Motor.

Therefore, with the additional relation that $i_a = i_f$ the differential equations are:

$$V_a = i_a(R_a + R_f) + L_a \frac{di_a}{dt} + L_f \frac{di_a}{dt} + K_b i_a \omega_m \quad \textbf{(15)}$$

$$T_e = K_b * i_a^2 = T_l + B\omega_m + J\frac{d\omega_m}{dt} \quad \textbf{(16)}$$

The dynamic equations for simulation are:

$$\dot{i}_a = \frac{V_a}{L_a + L_f} - \frac{(R_a + R_f)}{(L_a + L_f)} i_a - \frac{K_b \omega_m}{L_a + L_f} i_a \quad \textbf{(17)}$$

$$T_e = K_b * i_a^2 \quad \textbf{(18)}$$

$$\dot{\omega}_m = \frac{1}{J}(T_e - B\omega_m - T_l) \quad \textbf{(19)}$$

The SIMULINK® model for the Series Wound D.C. Motor and the sub-models of each of the subsystems are shown in Figure 6 below.

TRANSIENT PERFORMANCES

Machine Data

The motor parameters of Table 1 are used for the simulations.

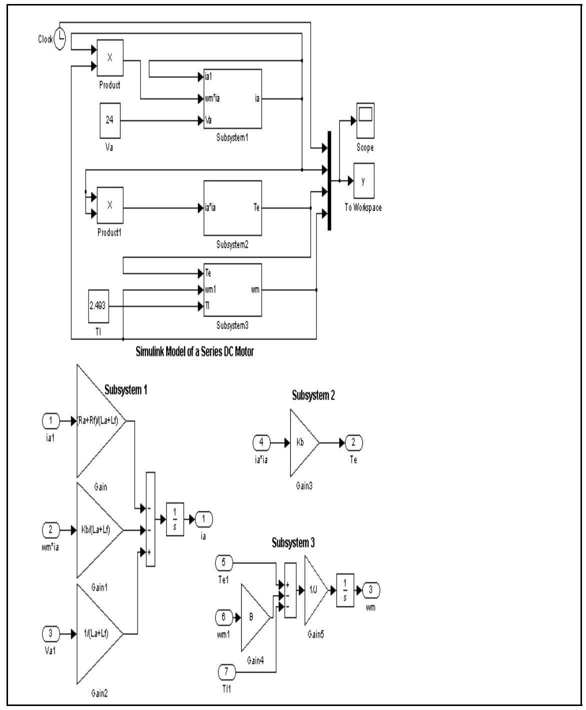

Figure 6: SIMULINK and Sub-system Models of Series Wound D.C. Motor.

Table 1: Motor Parameters.

	SEDM/ SDM	SWDM
Ra	0.0130Ω	0.0130Ω
La	0.010H	0.010H
R_f	1.430Ω	0.0260Ω
L_f	0.1670H	0.1670H
J	0.210Kg-m^2	0.210Kg-m^2
B	1.074e-6Nms^2	1.074e-6Nms^2
T_L	2.4930Nm	2.4930Nm
K_b	0.004N-m/A^2	0.004N-m/A^2
Va	24V	24V
V_f	12V	24V

Where:
SEDM-Separately Excited D.C. Motor
SDM-Shunt D.C. Motor
SWDM-Series Wound D.C. Motor

Response Curves

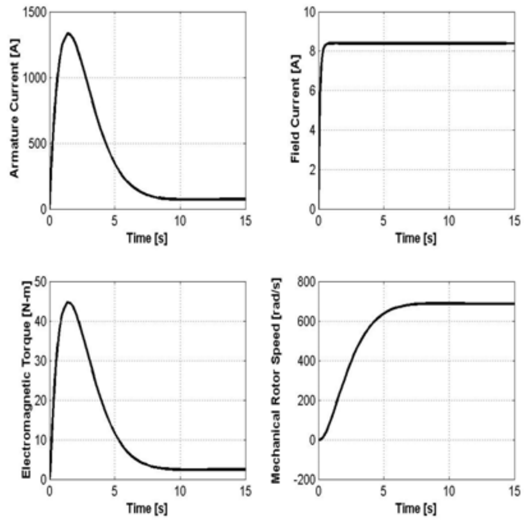

Figure 7: Startup Characteristics of a Separately Excited D.C. Motor.

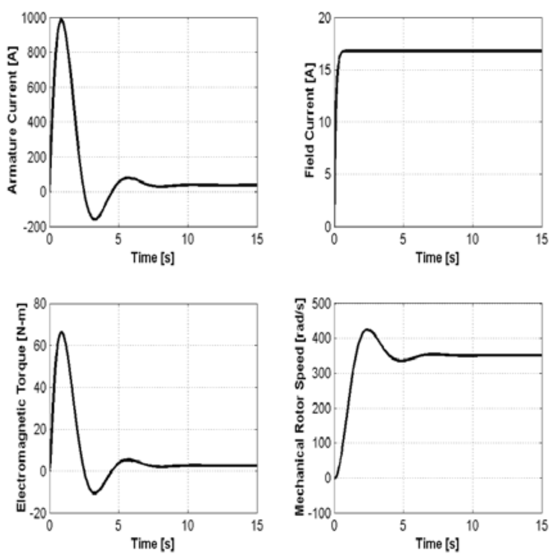

Figure 8: Startup Characteristics of a Shunt D.C. Motor.

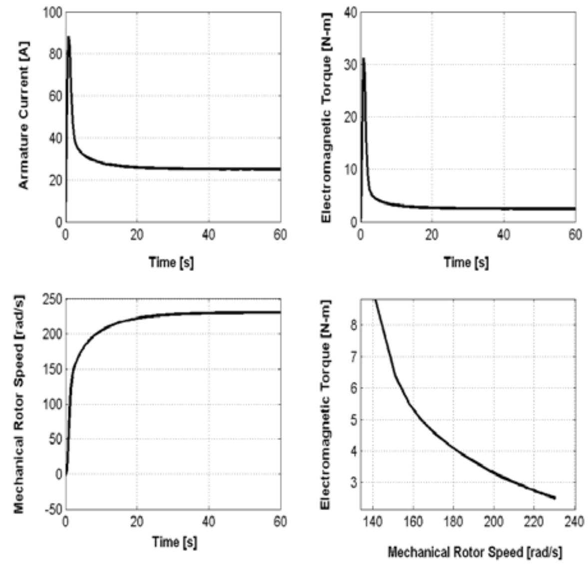

Figure 9: Startup Characteristics of a Series Wound D.C. Motor.

State Space Representation

The state space representation of an n^{th} order systems described by linear differential equations is [7];

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

where:

x=State variable vector
u=Input vector
A=Coefficient matrix or System matrix
B=Input or Control or Driving matrix
y=Output vector
C=Output matrix
D=Transmission matrix

The MATLAB® command:
[A, B, C, D]=linmod('model_name') is used to extract the linear models of the SIMULINK® systems.

Using load torque (T_l) as input and electromechanical speed (ω_m) as output, the state space representation of the three models are realized as shown below.

(a) Separately Excited D.C. Motor

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.0 & 0.00 & 0.00 \\ 0.00 & -1.3 & 0.00 \\ 0.00 & 0.00 & -8.5629 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -4.7619 \\ 0.0000 \\ 0.0000 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.00 & 0.00 & 0.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(b) Shunt D.C. Machine

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -0.0 & 0.00 & 0.00 \\ 0.00 & -1.3 & 0.00 \\ 0.00 & 0.00 & -8.5629 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -4.7619 \\ 0.0000 \\ 0.0000 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.00 & 0.00 & 0.00 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(c) Series Wound D.C. Motor

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.000 & 0.0000 \\ 0.0000 & -0.2203 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} -4.7619 \\ 0.0000 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1.0000 & 0.0000 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Controllability and Observability

A system is said to be controllable if the determinant of the Controllability Matrix is not zero and observable if the determinant of the Observability Matrix is not zero. Controllability Matrix and Observability Matrix can, respectively, be realized in MATLAB® as **ctrb(A,B)** and **obsv(A,C)** and their determinants obtained as **det(ctrb(A,B))** and **det(obsv(A,C)),** respectively.

It is observed that **det(ctrb(A,B)) = 0 and det(obsv(A,C)) = 0** for the three machine models. Thus the systems are neither controllable nor observable. However, a controllable and observable system can be obtained by selecting different motor parameters other than speed and electromechanical torque.

Stability Studies

A system is said to be stable if the real parts of the eigenvalues are all negative. For a system represented in state space, the eigenvalues are determined in MATLAB® as **eig(A).** The eigenvalues are determined for the machine models as shown below.

(a) Separately Excited D.C. Motor
eig(A)=-8.5629; -1.3000;-0.0000

(b) Shunt D.C. Motor
eig(A)=-8.5629; -1.3000;-0.0000

(c) Series D.C. Motor
eig(A)=-0.2203;-0.000

It can be seen that all the eigenvalues have negative real parts; the three systems are, therefore, stable.

CONCLUSION

The work has sufficiently demonstrated how elegant SIMULINK®, in combination with MATLAB® codes, can be for purposes of modeling and analysis of D.C. Machine transient behaviors.

The SIMULINK® models developed are very useful tools for teaching of D.C. machines' behaviors under transient conditions. Students of Electrical Machines will benefit immensely from this study as the block-based approach of analysis adopted is simple, elegant and easy to comprehend as opposed to time-wasting program writing which several authors have used in the past.

REFERENCES

1. Slemon, G.R. and Straughen, A. 1982. *Electric Machines*. Addison Wesley Publishing Company: New York, NY.

2. Ryff, D., Platnick, D., and Karnas, J. 1987. *Electric Machines and Transformers, Principles and Application.* Prentice-Hall Inc.: New York, NY.

3. Wildi, T. 2000. *Electric Machines, Drives, Power Systems*. Prentice-Hall Inc.: New York, NY.

4. Ostovic, V. 1994. *Computer Aided Analysis of Electric Machines*. Prentice Hall International Ltd.: London, UK.

5. Guru, B.S. and Hiziroglu, H.R. 1998. *Electric Machines and Transformer*. Oxford University Press: London, UK.

6. Okoro, O.I. 2005. "Application of Numerical Software in Electrical Machine Modeling". *LAUTECH Journal of Engineering and Technology.*

7. Ogata, K. 2003. *Modern Control Engineering, Fourth Edition.* Pearson Education Press: New York, NY.

ABOUT THE AUTHORS

Dr.-Ing. O.I. Okoro received his B.Eng. and M.Eng. degrees in Electrical Engineering from the University of Nigeria. He holds a Ph.D. in Electrical Machines from the University of Kassel, Germany under the DAAD scholarship program. He is a registered Electrical Engineer (COREN) and a corporate member of the Nigerian Society of Engineers (MNSE) and the IEEE (MIEEE). He currently serves as the Head of the Department of Electrical Engineering at the University of Nigeria, NSUKKA.

Engr. C.U. Ogbuka graduated from the Department of Electrical Engineering, University of Nigeria, Nsukka with First Class Honors. He was the best graduating student during the 2002/2003 academic session. He is currently pursuing his M.Eng. degree in Power Devices. His research interest is in Electric drives and Power Electronics.

Dr. M.U. Agu holds a Ph.D. in Power Electronics from the University of Toronto, Canada. He is an Associate Professor in the Department of Electrical Engineering, University of Nigeria, Nsukka. His research interests include power electronics and the control of electric drives. He is a member of the NSE and IEEE.

SUGGESTED CITATION

Okoro, O.I., C.U. Ogbuka, and M.U. Agu. 2008. "Simulation of D.C. Machines Transient Behaviors: Teaching and Research". *Pacific Journal of Science and Technology*. 9(1):142-148.

Pacific Journal of Science and Technology