The Effects of Friction Factors on Capillary Tube Length

M.A. Akintunde, Ph.D.

Department of Mechanical Engineering, Federal University of Technology, P.M.B. 704, Akure, Nigeria.

E-mail: ajyinka@yahoo.com

ABSTRACT

Capillary tubes are one of the major components of Vapor Compression Refrigeration Systems. Research efforts have shown that some parameters such as friction factor, dryness fraction, and Reynolds number affect the required length and diameter of a capillary tube for a given refrigeration capacity. Consequently, many friction factor models were developed by researchers based on these parameters. Using these developed friction factor models in the estimation of capillary tube lengths yielded different results. It was discovered that those friction factors that are based on the dryness fraction yielded better results as compared to those in the American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) Handbook.

The discrepancy observed from those friction factors that are based on the dryness fraction ranges between 0.05% and 0.85% while others range between 0.5% and 1.9% above that of ASHRAE under the same conditions. Furthermore, both McAdams’ and Duckler’s equations for two-phase viscosity were employed so that the deviation in the estimated lengths could be compared. The tube lengths generated by combining various friction factor models with McAdams’ equation are much closer to that of ASHRAE standard than those of Duckler’s equation. On the average, the estimated lengths using McAdams’ and Duckler’s equations exceeded ASHRAE standard by 1.65% and 4.13% respectively.

(Keywords: capillary, refrigeration, friction factor, tube length, mathematical model, thermodynamic properties, pressure drop)

INTRODUCTION

The most widely used cycle in the field of refrigeration and air–conditioning is the vapor-compression cycle. In this cycle, vapor is compressed then condensed to liquid form; following this, the pressure is dropped so that fluid can evaporate at low pressure. This cycle is performed by four major components, namely: the compressor, condenser, evaporator, and expansion device. Each of these components has its own peculiar behavior, and each component is influenced by conditions imposed by the other members of the quartet (Stoecker and Jones, 1982).

The influence of one component on the other results in changes in the thermal properties of the working fluid. As a result, balanced points must be reached between these components for effective performance. Stoecker and Jones (1982) reported on balanced points for the first components, while Akintunde (2004) extends the work to cover the last component; the capillary tube. A mathematical model for the generation of capillary tube length for both R12 and R22 was developed based on the thermodynamic properties of these refrigerants this model was used to develop a computer program (REF-2004), (Akintunde, 2004).

One of the major components of the vapor compression refrigeration systems is the expansion device. This serves the following purposes: reduces the pressure of liquid refrigerant; regulates the flow of refrigerant to the evaporator; and maintains the minimum possible pressure in the condenser at which all the refrigerant can condense without choke flow (Stoecker and Jones, 1982). The most common examples of expansion devices are the capillary tube and superheat controlled expansion valves. Other examples include the float valve and the constant pressure expansion valve.

Liquid refrigerant enters the expansion device and as it flows through, the pressure drops because of friction and acceleration of the refrigerant. As a result, some refrigerants flash...
into vapor. The amount of refrigerant that flashes into vapor depends largely on the friction factor, tube diameter, and flow rate (Jung et al., 1999). It was later discovered that the appropriate length of capillary tube depends on these three factors, in order to prevent choke flow and at the same time enhance system performance, (Chen et al., 2000).

The capillary tube, although physically simple, is behaviorally complex. Some items of complex mathematical analysis include the friction factor for the two phase flow, the thermal contact with the suction line, the metastable flow, and the oil circulation with the refrigerant, to mention a few.

Many researchers have investigated the flow through capillary tubes and a review on the subject can be found in the works of Melo et al., (1988); Schultz, (1985); Stoecker and Jones, (1982); Jung et al. (1999), and Akintunde (2004).

In order to model a uniform length of capillary tube for a given refrigeration capacity under a stated working condition, many researchers had presented a series of friction factors. A review of these friction factors can be found in the works of Jung et al. (1999) and Akintunde (2004). Most of these friction factors when tested were found to generate different capillary tube lengths compared with those in American Society of Heating, Refrigerating and Air-Conditioning Engineers (ASHRAE) handbook (2002).

In this article, another mathematical model for the estimation of capillary tube length under a predetermined condition of operation was developed. The model was a modification of Jung et al. (1999) and Stoecker and Jones (1982) models. The new model was based on thermodynamic properties of R12 and R22, while those of Jung et al. and Stoecker and Jones were based on R12, R22, and their alternatives respectively.

The mathematical model was used to develop a computer program which was used to generate data for the friction factor. The program was then used to estimate capillary tube lengths using the newly developed friction factor and the friction factors of other researchers. The estimated lengths were then compared with those in ASHREA handbook under a stated working condition.

**CAPILLARY TUBE MODEL**

Figure 1 shows the representation of the capillary tube used for this modeling. For the analysis the following assumptions were made: the mass flow rate ($\dot{m}$) is constant and adiabatic conditions prevailed in the capillary tube (Kim, et al., 2002; Wijaya, 1992; Chang and Ro, 1996; ASHRAE handbook (2002)).

![Schematic Diagram of a Capillary Tube](http://www.akamaiuniversity.us/PJST.htm)

**Figure 1**: Schematic Diagram of a Capillary Tube.

At steady state, since the mass flow rate ($\dot{m}$) was assumed to be constant therefore the mass flow equation can be written as shown in Equation (1) (the cross-sectional area was assumed to be constant).

$$\dot{m} = \frac{V_1 A}{v_1} = \frac{V_2 A}{v_2} \quad (1a)$$

or

$$\dot{m}/A = G = \frac{V_1}{v_1} = \frac{V_2}{v_2} \quad (1b)$$

Generally Equation (1b) can be modified to give Equation (2).

$$V = Gv \quad (2)$$

Since an adiabatic condition was assumed in the capillary tube, then an energy equation can be written as shown in Equation (3), for a flow process (Jung et al., 1999), while the momentum equation is given by Equation (4) (Akintunde, 2004).
To determine dryness fraction (κ) Equations (2), (7), and (8) were combined with Equation (3), these resulted in Equation (19). 

\[ V_m = \frac{V_1 + V_2}{2} \]  

Equation (3) was solved for κ₂ [κ₁, is assumed to be zero- saturated or subcool liquid leaving the condenser] the dryness factor at the state point (2). For the purpose of the computer modeling the program will be terminated at the point where the Mach number becomes unity. Hence for the dynamic states, the Mach number (Mφ) as expressed by Jung et al. (1999) (Equation 20a and b) was modified by Akintunde (2004) for steady state as shown in Equation (21).

\[ M_\phi = \left\{ G^2 \left[ x \frac{dv}{dp} + (1-x) \frac{dv_f}{dp} + v_{fl} \left( \frac{dx}{dp} \right)_b \right] \right\}^{1/2} \]  

where:

\[ \phi = \left[ 1 + G^2 \frac{v_{fl} (V_f + xV_{fl})}{h_{fl}} \right] \]  

\[ M_\phi = \left\{ G^2 \left[ x \frac{v_{fl} (V_f + xV_{fl})}{P_1 - P_2} + (1-x) \frac{v_{fl} (V_f - V_{fl})}{P_1 - P_2} + v_{fl} \left( \frac{x_2 - x_1}{P_1 - P_2} \right)_b \right] \right\}^{1/2} \]  

The property equations for R12 and R22 can be obtained from the works of Jung et al. (1999); Stoecker and Jones (1982); and Akintunde (2004). The flow chart for the computer program is shown in Figure 2.

**METHODOLOGY**

The friction factor equations were tested under varied conditions as follows, using 10 kW capacity refrigerator:

(i) Saturated condition of 40 C condensing temperature, 0.01 kg/s flow rate; 1.62 mm; capillary tube internal diameter and –5 C evaporation temperatures. The ambient temperature was assumed constant at 35 C.
Table 1: Friction Factor Equations (Source: Jung et al., 1999 and Akintunde, 2004).

<table>
<thead>
<tr>
<th>Code</th>
<th>Author</th>
<th>Friction Factor Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>F1</td>
<td>Stoecker</td>
<td>[ f = \frac{0.33}{Re^{0.25}} ] (9)</td>
</tr>
<tr>
<td>F2</td>
<td>Blausius</td>
<td>[ f = \frac{0.3}{Re^{0.25}} ] (10)</td>
</tr>
<tr>
<td>F3</td>
<td>Goldstein</td>
<td>[ f = 0.02 ] (11)</td>
</tr>
<tr>
<td>F4</td>
<td>Erth</td>
<td>[ f = \frac{3.1}{Re^{0.5}} \text{Exp}\left(\frac{1.0 - x^{0.25}}{2.4}\right) ] (12)</td>
</tr>
<tr>
<td>F5</td>
<td>Sami</td>
<td>[ f = \frac{3.1}{[Re(1-x)]^{0.5}} \text{Exp}\left(\frac{1.0 - x^{0.25}}{2.4}\right) ] (13)</td>
</tr>
<tr>
<td>F6</td>
<td>Pate et al</td>
<td>[ f = \frac{3.49}{Re^{0.47}} ] (14)</td>
</tr>
<tr>
<td>F7</td>
<td>Hopkins</td>
<td>[ f = \frac{0.217}{Re^{0.2}} ] (15)</td>
</tr>
<tr>
<td>F8</td>
<td>Jung et al</td>
<td>[ f = \frac{0.25}{Re^{0.27}} ] (16)</td>
</tr>
<tr>
<td>F9</td>
<td>Present work</td>
<td>[ f = \frac{0.25}{[Re(1-x)]^{0.28}} ] (17)</td>
</tr>
</tbody>
</table>

(ii) Varied subcooling temperatures. The subcooling temperature ranges between 2 C and 6 C in step of 0.5 C.

The generated tube lengths were compared with that of ASHRAE (which is 2.03 m) under the stated condition in (i) above. This was used as the benchmark for the comparison of the generated lengths when the various friction factors were combined with either McAdams’ or Duckler’s equations for two-phase viscosity. These two equations were judged the best for two phase flow by Jung et al. (1999). Each of the friction factors combined with either McAdams’ or Duckler’s equations for two-phase viscosity were used in turn to generate data under various conditions using the developed computer program. A new friction factor equation (F9) was then developed by fitting data into Equation (22) using dimensional analysis.

\[ f = \phi(Re, x, d, v) \] (22)

RESULTS AND DISCUSSIONS

The generated lengths when each of the friction factor equation was combined with the McAdams’ equation for two-phase viscosity was denoted by \( L_M \) while the combination of the friction factor equations with Duckler’s equation was tagged \( L_D \). \( L_A \) stands for the corresponding length given by ASHRAE handbook under the same conditions.

Figure 3 shows the generated tube lengths under the saturated conditions, while Figures 4 to 8 show the corresponding lengths under various conditions of subcooling. Figure 9 shows the summary of the findings.
Figure 2: Flow Chart for Capillary Tube Model.

VERGE = Refrigerant properties subroutine
T1 = Evaporating temperature
T2 = Condensing temperature
y = Dryness fraction
h = Enthalpy
v = humid volume
mu = Viscosity factor
V = Velocity
Re = Reynolds number
f = Friction factor
fm = Mean friction factor
Vm = Mean velocity
vm = mean viscosity
L = Length
dl = Change in length
MQ = Mach number
R = Refrigerant

The Pacific Journal of Science and Technology
http://www.akamaiuniversity.us/PJST.htm

Volume 8. Number 2. November 2007 (Fall)
Figure 3: Capillary Tube Lengths at Saturated Points.

Figure 4: Capillary Tube Length at 2 C Subcooling.

Figure 5: Capillary Tube Length at 3 C Subcooling.

Figure 6: Capillary Tube Length at 4 C of Subcooling.

Figure 7: Capillary Tube Length at 5 C Subcooling.

Figure 8: Capillary Tube Length at 6 C Subcooling.
Figure 3 shows that under the saturated condition, both $L_M$ and $L_D$ generated by the use of the friction factors are longer than that of ASHREA except for F7. A critical look shows that $L_M$ (that is, the length generated by McAdams’ equation for two-phase viscosity with the various friction factors) are much closer to that of ASHRAE than the corresponding $L_D$.

As could be observed in Figures 4 to 8, as the degree of subcooling increases, the various length approaches stipulated by ASHRAE change. Figure 9 also justifies this, which indicates that the tube length may likely converge at higher degree of subcooling.

These friction factors were now grouped into three groups: A; B and C.

(A) Those friction factors that dependent on the Reynolds number and the dryness fraction. These are F4, F5 and F9.

(B) Those friction factors that depend on the Reynolds number alone. These are: F1; F2; F6; F7 and F8.

(C) Those friction factors that neither depends on the Reynolds number nor dryness fraction (F3).

Figures 3 to 9 show very clearly that group A, (that is, Figure 4 and 5), gives outrageous lengths as compared with the corresponding ASHRAE standard length, while F9 gives lengths that are very closed to that of the ASHRAE (that is $L_A$). It should be noted that both Figure 4 and 5 are given as exponential function of dryness fraction, while Figure 9 was not. From the same figures and Figure 1, 2, 6, 7 and 8 (group B) generated lengths that are much closer to that of ASHRAE as compare to Group A.

From those figures also, it could be observed that F3 generated constant length (both $L_D$ and $L_M$), which could be expected since F3 is a constant. More so, it is observed that both $L_M$ and $L_D$ are longer than $L_A$. Judging from the lengths generated by group C, it is clear that the friction factor could not be given as a constant term, while the observation from group A (without F9) predicted that the friction factor may not depends, so much, on the dryness fraction and the Reynolds number. This is because the lengths generated by group C are closer to the benchmark than those of group A.

Considering the lengths generated by Figure 9, (the newly developed friction factor equation), it shows that the required capillary tube length depends on both Reynolds number ($Re$) and dryness fraction ($x$). This shows further that the dependency of friction factor on Re and x is not exponential.

CONCLUSION

The present study examined the generated capillary tube lengths based on friction factors and viscosity equations for two-phase flow, which is prevalent in the capillary tube. The lengths generated by various friction factors under stated conditions were compared with the standard lengths given by ASHRAE. It was clearly shown that the required capillary tube length for a specified condenser condition depends on both Reynolds number ($Re$) and dryness fraction ($x$) and not on either alone. It was also noted in the study that the required relationship between these two factors (that $Re$ and $x$) should not be in exponential form. It was shown that the generated lengths approach the ASHRAE requirement as the degree of subcooling is increased. A new friction factor was then developed from the generated data. This friction factor developed was tested along with the others under the same conditions and was found to yield better-required length as compared with ASHRAE standard.

NOMENCLATURE AND SUBSCRIPTS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Surface or cross sectional area ($m^2$)</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Diameter (m)</td>
</tr>
</tbody>
</table>
f Friction factor
G Mass flux (kg/m²)
L Length required (m)
ΔL Length increment (m)
m Mass rate of flow (kg/s)
M Mach number
P Pressure (N/m²) (or Power (W))
Re Reynolds number
T Temperature (K)
T_a Ambient temperature (K)
v Velocity (m/s)
κ Dryness fraction
μ Dynamic viscosity (Pa.s)
ν Humid volume (m³/kg)
ρ Density (kg/m³)
fg mixture
g gas (or vapor)
1 Inlet
2 Outlet
m mean value
A ASHRAE
M McAdams’ combination
D Duckler’s combination

ACKNOWLEDGEMENTS

The author hereby acknowledges and thanks the Federal University of Technology, Akure for providing the grant for this research. The grant reference number is URGC/MINOR/2007/183.

REFERENCES


ABOUT THE AUTHOR

Dr. M. A. Akintunde, graduated from Obafemi Awolowo University, Ile-Ife in 1992 with first class honors in the Department of Mechanical Engineering. He studied Building Services Engineering at The Federal University of Technology, Akure (Nigeria). He completed his Master’s degree in 1997 and Ph.D. in 2004. He is a Senior Lecturer in Mechanical Engineering at the Federal University of Technology, Akure Nigeria. His research interest includes refrigeration and air-conditioning, energy systems, thermodynamics, and mechanics.

SUGGESTED CITATION