Positive Numbers Divisible by their Iterative Digit Sum Revisited.

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ABSTRACT

This paper is about the existence of numbers divisible by their iterative digit sum (NDIDS). Three conjectures were proposed, firstly, 9 multiply a NDIDS produces a multiple NDIDS. Secondly, there are no 15 or more consecutive NDIDS in the sequence of positive integers. Lastly, all Niven numbers are also NDIDS but the converse is not true. The detailed composition, properties, sequences and comparison with Niven numbers of NDIDS are discussed.

(Keywords: iterative digit sum, digital root sequences, Nivens numbers, NDIDS)

INTRODUCTION

Iterative digit sum or digital root or repeated digital sum is the continual adding up of digits of non-negative integers until a single digit is obtained. The iteration implies that the results of the previous digit sum be used repeatedly until a single digit is obtained, this is termed additive persistence.

For example, the iterative sum of digit of 788

$$788 = 7 + 8 + 8 = 23 = 2 + 3 = 5$$

The most interesting thing about the iterative sum of digit is that it is equivalent to modulo 9. That is when a non-negative integer is divided by 9; the integer remainder and the iterative digits sum are the same.

For example, $87 = 106_9$, $8+7=15$, $1+5=6$

There is also a shorter method of finding the iterative digit sum (IDS) or digital root (DR) of non-negative integers using the formula:

$$IDS_n = 1 + ((n-1) \bmod 9) \quad \textbf{(1)}$$

For example, The IDS of 78 is computed as:

$$\begin{aligned} 1 + (78-1) \bmod 9 &= 1 + (77 \bmod 9) \\ &= 1 + 5 = 6 \end{aligned}$$

The nature of iterative digit sum is mainly on the digit sum or the Hamming weight and can be extended to different number bases (see [1-3], for details on iterative digit sum).

The aim of this paper is to find positive numbers divisible by their iterative digit sum and compare them with the Niven numbers which are positive integers divisible by their sum of digit. Divisibility here means that the outcome of the division of positive integers by their respective iterative digit sum must be a whole number. This numbers is termed numbers divisible by their iterative digit sum (NDIDS) The result will be compared with the digit and iterative digit sum of positive root of integers [4-5], hence a number divisible by its digit sum may not be divisible by their iterative digit sum or vice versa or both.

The motivation arises from similar results obtained from the digit and iterative digit sum of Sophie Germain and safe primes [6], palindromic numbers, repdigits and repunit numbers [7], and Fibonacci numbers and their powers [8].

Existence of NDIDS and multiple NDIDS

The following examples show the existence of numbers divisible or not divisible by their iterative digit sum and consequently multiple numbers divisible by their iterative digit sum.

Example 1.

78 is a NDIDS, $\frac{78}{7+8}=\frac{78}{15}=\frac{78}{1+5}=\frac{78}{6}=13$

Example 2.

187 is not a NDIDS,

$$\frac{187}{1+8+7}=\frac{187}{16}=\frac{187}{1+6}\neq \text{ whole number}$$

Example 3.

56 is a 2-NDIDS,

$$\frac{56}{11}=\frac{56}{2}=28 \qquad \frac{28}{10}=\frac{28}{1}=28$$

Example 4.

504 is a 3-NDIDS, $\frac{504}{9}=56$, repeat the process of Example 3.

Example 5.

4536 is a 4-NDIDS, $\frac{4536}{18}=\frac{4536}{9}=504$ repeat the processes of example 4 and 3.

Note that $56\times 9=504,\ 504\times 9=4536.$

We conjecture that 9 multiples a NDIDS produces a multiple NDIDS.

That is, given a NDIDS, then:

$9\times$1-NDIDS=2-NDIDS, $9\times$2-NDIDS=3-NDIDS and so on. Surprising, this is one of the properties of the sum of digits of positive integers.

Numbers Divisible by their Iterative Digit Sum

The first 30 positive integers divisible by their iterative digit sum are listed in Table 1, along with their iterative digit sum and constant.

We also conjure that by close observation of the data, that there are no 15 or more consecutive NDIDS in the sequence of positive integers.

The number of NDIDS per hundreds showed that at least 50% of positive integers per 100 are NDIDS. The first 10 hundreds are shown in Table 2.

Table 1: The First 30 Numbers Divisible by their Iterative Digit Sum.

Positive integers	Iterative digit sum	Constant
1	1	1
2	2	1
3	3	1
4	4	1
5	5	1
6	6	1
7	7	1
8	8	1
9	9	1
10	1	10
12	3	4
18	9	2
19	1	19
20	2	10
21	3	7
24	6	4
27	9	3
28	1	28
30	3	10
36	9	4
37	1	37
38	2	19
39	3	13
40	4	10
42	6	7
45	9	5
46	1	46
48	3	16
50	5	10
54	9	6

Table 2: The Frequency of NDIDS per Hundred for the First 1,000 Positive Integers.

Range	Frequency
1-100	56
101-200	52
201-300	53
301-400	52
401-500	51
501-600	54
601-700	51
701-800	53
801-900	51
901-1000	54

Some Sequences Generated

The numbers divisible by their iterative digit sum formed a sequence A of positive integers. The sequence is given as;

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 19, 20, 21, 24, 27, 28, 30, 36, 37, 38, 39, 40, 42, 45, 46, 48, 50, 54, 55, 56, 57, 60, 63, 64, 66, 70, 72, 73, 74, 75, 76, 78, 80, 81, 82, 84, 90, 91, 92, 93, 95, 96, 99, 100, … (A)

All the single units are elements of the sequence. Secondly, not all multiples of 10 are NDIDS, for example; This can be represented in a sequence;

130, 160, 170, 250, 260, 310, 340, 350, 430, 490, 520, 530, 610, 620, 670, 710, 790, 850, 880, 890, 970, 980, … (B)

The following positive integers are not NDIDS which formed the sequence C. This is the complement of sequence A, but the domain is the positive integer line.

11, 13, 14, 15, 16, 17, 22, 23, 25, 26, 29, 31, 32, 33, 34, 35, 41, 43, 44, 47, 49, 51, 52, 53, 58, 59, 61, 62, 65, 67, 68, 69, 71, 77, 79, 83, 85, 86, 87, 88, 89, 94, 97, 98, … (C)

The following NDIDS have square numbers as the constant obtained from their computation, that is positive integers divided by their iterative digit sum and leaving a whole number as a constant.

1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 24, 36, 48, 64, 75, 81, 96, 100, 128, 144, 147, 150, 192, 200, 225, 256, 289, 294, 300, … (D)

The following NDIDS does not have square numbers as the constant obtained from their computation, which are positive integers divided by their iterative digit sum and leaving a whole number as a constant.

10, 18, 19, 20, 21, 27, 28, 30, 37, 38, 39, 40, 42, 45, 46, 50, 54, 55, 56, 57, 60, 63, 66, 70, 72, 73, 74, 76, 78, … (E)

The following NDIDS have cube numbers as the constant obtained from their computation, that is positive integers divided by their iterative digit sum and leaving a whole number as a constant.

1, 2, 3, 4, 5, 6, 7, 8, 9, 64, 72, 128, 192, 243, 256, 320, 343, 384, 448, 512, 576, 686, 1000, 1029, 1125, … (F)

The following NDIDS does not have cube numbers as the constant obtained from their computation, which are positive integers divided by their iterative digit sum and leaving a whole number as a constant.

10, 12, 18, 19, 20, 21, 24, 27, 28, 30, 36, 37, 38, 39, 40, 42, 45, 46, 48, 50, 54, 55, 56, 57, 60, 63, 66, 70, … (G)

The following NDIDS have numbers raised to the power of four as the constant obtained from their computation, that is positive integers divided by their iterative digit sum and leaving a whole number as a constant.

1, 2, 3, 4, 5, 6, 7, 8, 9, 48, 96, 144, 729, 768, … (H)

The following NDIDS does not have numbers raised to the power of four as the constant obtained from their computation, that is positive integers divided by their iterative digit sum and leaving a whole number as a constant

10, 12, 18, 19, 20, 21, 24, 27, 28, 30, 36, 37, 38, 39, 40, 42, 45, 46, 50, … (I)

The following Palindromic numbers are NDIDS. The sequence is given as:

1, 2, 3, 4, 5, 6, 7, 8, 9, 55, 66, 99, 111, 171, 181, 222, 252, 262, 272, 282, 292, 333, 343, 363, 414, 424, 434, 444, 474, 505, 525, 545, 585, 595, 606, 616, 636, 656, 666, 676, 686, 696, 747, 757, 777, 828, 838, 848, 858, 868, 888, 909, 919, 939, 999, … (J)

The following Repdigit numbers are NDIDS. The sequence is given as:

1, 2, 3, 4, 5, 6, 7, 8, 9, 55, 66, 99, 111, 222, 333, 444, 666, 777, 888, 999, … (K)

The following primes are NDIDS. The sequence is given as:

2, 3, 5, 7, 19, 37, 73, 109, 127, 163, 181, 199, 271, 307, 379, 397, 433, 487, 523, 541, 577, 613, 631, 739, 757, 811, 829, 883, 919, 937, 991, … (L)

The following primes are not NDIDS. The sequence is given as:

11, 13, 17, 23, 29, 31, 41, 43, 47, 53, 59, 61, 67, 71, 79, 83, 89, 97, 101, 103, 107, 113, 131, 137, 139, 149, 151, 157, 167, 173, 179, … (M)

The following odd numbers are NDIDS. The sequence is given as:

1, 3, 5, 7, 9, 19, 21, 27, 37, 39, 45, 55, 57, 63, 73, 75, 81, 91, 93, 95, 99, … (N)

The following even numbers are NDIDS. The sequence is given as:

2, 4, 6, 8, 10, 12, 18, 20, 24, 28, 30, 36, 38, 40, 42, 46, 48, 50, 54, 56, 60, … (O)

In addition, the number of both even and odd numbers that are NDIDS per hundred in the first 1000 integers is shown in Table 3. Clearly, NDIDS even numbers are more than NDIDS odd numbers.

Table 3: Frequency of even and odd NDIDS per hundred in the first 1000 positive integers.

Range	Even	Odd
1-100	35	21
101-200	34	18
201-300	34	19
301-400	33	19
401-500	34	17
501-600	35	19
601-700	33	18
701-800	34	19
801-900	33	18
901-1000	34	20

The iterative digit sum divides the positive integers into 9 distinct classes or division by 9 (modulo 9). Consequently, nine distinct sequences are obtained, by grouping the iterative digit sum of numbers divisible by their iterative digit sum. The sequences are as follows:

The following NDIDS have 1 as their respective iterative digit sum.

1, 10, 19, 28, 37, 46, 55, 64, 73, 82, 91, 100, 109, 118, 127, 136, 145, 154, 163, 172, 181, 190, 199, … (P)

The numbers comprises of many primes.

The following NDIDS have 2 as their respective iterative digit sum.

2, 20, 38, 56, 74, 92, 110, 128, 146, 164, 182, 200, 218, 236, 254, 272, 290, 308, 326, 344, 362, … (Q)

The numbers are 2 multiply the numbers of sequence P. The numbers are all multiples of 2.

The following NDIDS have 3 as their respective iterative digit sum.

3, 12, 21, 30, 39, 48, 57, 66, 75, 84, 93, 102, 111, 120, 129, 138, 147, 156, 165, 174, 183, 192, … (R)

The numbers are all multiples of 3.

The following NDIDS have 4 as their respective iterative digit sum.

4, 40, 76, 112, 148, 184, 220, 256, 292, 328, 364, 400, 436, 472, 508, 544, 580, 616, 652, 688, 724, 760,… (S)

The numbers are 4 multiply the numbers of sequence P or 2 multiply the numbers of sequence Q. The numbers are all multiples of 2 and 4.

The following NDIDS have 5 as their respective iterative digit sum.

5, 50, 95, 140, 185, 230, 275, 320, 365, 410, 455, 500, 545, 590, 635, 680, 725, 770, 815, 860, 905, 995, … (T)

The numbers are 5 multiply the numbers of sequence P. The numbers are all multiples of 5.

The following NDIDS have 6 as their respective iterative digit sum.

6, 24, 42, 60, 96, 114, 132, 150, 168, 186, 204, 222, 240, 258, 276, 294, 312, 330, 348, 366, 384, 402, 420, 438, 456, 474, … (U)

The numbers are 2 multiply the numbers of sequence R. The numbers are all multiples of 2, 3 and 6.

The following NDIDS have 7 as their respective iterative digit sum.

7, 70, 133, 196, 259, 322, 385, 448, 511, 574, 637, 700, 763, 826, 889, 952, … (V)

The numbers are 7 multiply the numbers of sequence P. The numbers are all multiples of 7.

The following NDIDS have 8 as their respective iterative digit sum.

8, 80, 152, 224, 296, 368, 440, 512, 584, 656, 728, 800, 872, 944, ... (W)

The numbers are 8 multiply the numbers of sequence P, or 4 multiply the numbers of sequence Q, or 2 multiple the numbers of sequence S. The numbers are all multiples of 2, 4 and 8.

The following NDIDS have 9 as their respective iterative digit sum.

9, 18, 27, 36, 45, 54, 63, 72, 81, 90, 99, 108, 117, 126, 135, … (Y)

The numbers are all multiples of 9.

Comparison with Niven Numbers

Niven or Harshad numbers are positive integers divisible by the sum of their digit. Comparison is important to compare divisibility by sum of digit and iterative sum of digit.

a). The frequency of both Niven numbers and NDIDS per hundred for the first 1000 positive integers is shown in Table 4.

Clearly Niven numbers are fewer than the numbers divisible by their iterative sum.

b). The first 10 positive integers are both Niven numbers and NDIDS.

Table 4: The number of NDIDS and Niven Numbers per Hundred for the First 1,000 Positive Integers.

Number Range	Niven numbers	NDIDS
1-100	33	56
101-200	26	52
201-300	25	53
301-400	22	52
401-500	22	51
501-600	20	54
601-700	16	51
701-800	18	53
801-900	17	51
901-1000	14	54

c). The following numbers are both NDIDS and Niven numbers and they formed a sequence given as:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 20, 21, 24, 27, 30, 36, 40, 42, 45, 48, 50, 54, 60, 63, 70, 72, 80, 81, 84, 90, 100, … (Z)

d). The following positive integers are neither NDIDS nor Niven numbers and they formed a sequence given as:

11, 13, 14, 15, 16, 17, 22, 23, 25, 26, 29, 31, 32, 33, 34, 35, 41, 43, 44, 47, 49, 51, 52, 53, 58, 59, 61, 62, 65, 67, 68, 69, … (AA)

e). The following positive integers are either NDIDS or Niven numbers and they formed a sequence given as:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 18, 19, 20, 21, 24, 27, 28, 30, 36, 37, 38, 39, 40, 42, 45, 46, 48, 50, 54, 55, 56, 57, 60, … (AB)

f). We also conjure that all Niven numbers are also NDIDS but the converse is not true. The following positive numbers are NDIDS but are not Niven numbers.

19, 28, 37, 38, 39, 46, 55, 56, 57, 64, 66, 73, 74, 75, 76, 78, 82, 91, 92, 93, 95, 96, 99, … (AC)

CONCLUSION

Examples were shown on the existence of numbers that are NDIDS, not NDIDS and multiple NDIDS, consequently, a conjuncture were proposed that 9 multiples a NDIDS produce a multiple NDIDS.

Also, another conjecture on whether there can be 15 or more consecutive NDIDS in the sequence of positive integers was suggested.

At least 50% of positive integers for every 100 numbers are NDIDS.

Unique sequences were obtained on NDIDS that have or does not have square, cube or numbers to power of 4, as the constant obtained from their computation.

Sequences of Palindromic, Repdigit, primes, odd and even numbers that are NDIDS were obtained. Odd NDIDS were found to be fewer than even NDIDS. Some relationships among the sequences of iterative digit sum of positive integers that are NDIDS were established.

Niven numbers were found to be fewer in numbers than NDIDS.

Finally, we conjure that all Niven numbers are also NDIDS but the converse is not true.

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