# Propagation of a Pressure Change in a Non-Viscous Fluid

**Edward O. Osagie, Ph.D.**

Geotech Services, Jackson, TN 38395

E-mail: eosagiee@yahoo.com*

## ABSTRACT

The propagation of a pressure change in a non-viscous fluid at rest is investigated from the point of view of continuum mechanics. The resulting pressure in a slightly disturbed fluid is also investigated. The velocity of the sound propagation obtained is in agreement with measured value.

(*Keywords:* pressure change, non-viscous fluid, sound propagation, continuum mechanics)

## INTRODUCTION

In a large number of practical applications, the density of gas varies very little if the pressure differences produced by the flow regime are small. Under this condition, the flow of gas around bodies or inside conduits may be studied by assuming that the gas density is constant. When the variations of pressure produced by the flow regime are large, the assumption of incompressible flow is no longer valid as the corresponding changes in density must be considered (Tolstoy, 1973).

Compressions and expansions of the gas involve work done on and by the gas, respectively, and these result in changes in temperature of the gas. Hence the laws of mechanics alone are insufficient for a complete description of the physical quantities involved. In addition, thermodynamics is necessary to completely determine these quantities (Pippard, 1980). This branch of science is sometimes called aerothermodynamics (Eckardt, 1960). In this paper, the propagation of a pressure change through a fluid is investigated using the laws of aerothermodynamics.

## PROBLEM FORMULATION

A fluid filling all of space and initially at rest has the following properties:

(i) it is incapable of conducting heat, but is compressible.

(ii) the stress tensor in the fluid is always:

$$T = -P\,I$$

where I is the identity tensor and P is the thermodynamic equilibrium pressure.

(iii) the temperature in the fluid may vary from place to place.

(iv) no body forces act.

Under these conditions, it is our objective to determine the resulting pressure if the velocity of the fluid is zero (that is, it is at rest).

## ANALYSIS

With the given assumptions, the equations governing the motion are:

Continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho\,\mathbf{U}) = 0 \qquad \textbf{(1)}$$

where ρ is the fluid density
t is time
U is the flow velocity vector field

Momentum

$$\rho \frac{d\,U}{dt} + \nabla P = 0 \qquad \textbf{(2)}$$

Entropy

$$\frac{dS}{dt} = 0 \qquad \textbf{(3)}$$

It will be convenient to obtain first an alternative form of Equation (3) containing the same variables as in Equation (1) and Equation (2). To do this, consider the equation of state:

$$P = P(\mho, S) \qquad \textbf{(4)}$$

Differentiating Equation (4), we obtain:

$$dP = \left(\frac{\partial P}{\partial \mho}\right)_S d\mho + \left(\frac{\partial P}{\partial S}\right)_\mho dS$$

$$\equiv X\, d\mho + Y\, dS \qquad \textbf{(5)}$$

Thus:

$$\frac{dS}{dt} = \frac{1}{Y}\left(\frac{dP}{dt} + X\frac{d\mho}{dt}\right) \qquad \textbf{(6)}$$

$$= \frac{1}{Y}\left(\frac{dP}{dt} - \frac{X}{\rho^2}\frac{d\rho}{dt}\right) \qquad \textbf{(7)}$$

Now,

$$\underset{v}{X} = -\left(\frac{\partial P}{\partial \mho}\right)_S = \rho^2\left(\frac{\partial P}{\partial \rho}\right)_S$$

If,

$$\frac{dS}{dt} = 0 \quad \text{implies that}$$

$$\frac{dP}{dt} = \left(\frac{\partial P}{\partial \rho}\right)_S$$

Let us define a variable $c^2$ ( P, ρ ) such that:

$$c^2 \equiv \left(\frac{\partial P}{\partial \rho}\right)_S$$

Then Equation (3) can be replaced by:

$$\frac{dP}{dt} = c^2\frac{dS}{dt} \qquad \textbf{(8)}$$

In steady state $U = 0$

So that.

$$\frac{dU}{dt} = 0 \quad \text{and} \quad \nabla\cdot U = 0$$

Hence from Equation (2):

$$\nabla P = 0 \qquad \textbf{(9)}$$

Thus pressure is constant throughout space but may vary with time, that is

$$P = P_0(t)$$

but from Equation (1):

$$\frac{d\rho}{dt} = 0 \quad \text{if } U = 0$$

so,

$$\frac{\partial P}{\partial t} = 0$$

Thus, P is independent of t also, that is;

$$P = \text{constant} \equiv P_0$$

Its density may still vary from place to place if its temperature does. Now if the fluid is slightly disturbed, its pressure will be $P_0 + P_1$ and its velocity will be:

U, both $P_1$ and U being small. Then:

$$\frac{\partial}{\partial t}(\rho_o + \rho_1) + \nabla\cdot[(\rho_o + \rho_1)U] = 0 \qquad \textbf{(10)}$$

$$(\rho_o + \rho_1)\frac{d\mathbf{U}}{dt} = -\nabla(P_o + P_1) \qquad \textbf{(11)}$$

$$\frac{\partial}{\partial t}(\rho_o + \rho_1) + U\cdot\nabla(\rho_o + \rho_1) \qquad \textbf{(12)}$$

$$= \frac{1}{C^2}\left[\partial(P_o + P_1) + U\cdot\nabla(P_o + P_1)\right]$$

Upon neglecting second order quantities we obtain:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad \textbf{(13)}$$

$$\rho_o \frac{\partial U}{\partial t} + \nabla P_1 = 0 \quad \textbf{(14)}$$

$$\frac{\partial \rho_1}{\partial t} + U \cdot \nabla \rho_o = \frac{1}{C^2}\frac{\partial P_1}{\partial t} \quad \textbf{(15)}$$

Subtract Equation (15) and Equation (13) to get:

$$-\rho_o \nabla \cdot U = \frac{1}{C^2}\frac{\partial P_1}{\partial t} \quad \textbf{(16)}$$

taking the derivative of Equation (16) with respect to t, we get:

$$-\rho_o \nabla \cdot \frac{\partial \mathbf{U}}{\partial t} = \left(\frac{1}{c^2}\right)\frac{\partial^2 P_1}{\partial t^2}$$

and using Equation (14), we get:

$$\frac{\partial^2 P_1}{\partial t^2} = c^2 \rho_o \nabla \cdot (\rho_o^{-1} \nabla P_1) \quad \textbf{(17)}$$

If $\rho_o$ = const, then $S = S_0$ = const

hence $c^2$ const, so Equation (17) becomes:

$$\frac{\partial^2 P_1}{\partial t^2} = C^2 \nabla^2 P_1$$

which is a wave equation.

Let us calculate c from the thermodynamic properties of the fluid.

We define:

$$K_s = -\frac{1}{\mho}\left(\frac{\partial \mho}{\partial P}\right)_s$$

where $K_s$ is the isentropic bulk modulus or incompressibility:

and $$K_\theta = -\frac{1}{\mho}\left(\frac{\partial \mho}{\partial P}\right)_\theta$$

where $K_\theta$ is the isothermal incompressibility

also $$\gamma \equiv \frac{c_p}{c_v}$$

where $\gamma$ is the ratio of specific heats

Now $$X = \rho^2 C^2 = \frac{\rho}{K_s}$$

so that $$C^2 = \frac{1}{\rho K_s}$$

evaluated in the initial state, that is:

$$C^2 = \frac{1}{\rho_o K_s (P_0, \theta_0)} \quad \textbf{(18)}$$

since $$K_s = \frac{K_\theta}{\gamma}$$

We may also write equ (18) in the form:

$$C^2 = \frac{\gamma (P_o, \theta_o)}{\rho_o K_\theta (P_o, \theta_o)} \quad \textbf{(19)}$$

Let us evaluate C for air at $0^0$C

Let $\theta_o, S_o, P_o, \mho_o = \frac{1}{\rho_o}$ describe some particular equilibrium state of an ideal gas

where the ratio of specific heats is $\gamma$

If $$\theta = \theta_o \text{ ( isothermal gas)}$$

$$\bar{R} = \frac{R}{M}$$

Where R is the gas constant and M the molecular weight ($\bar{R}$ appears instead of R in continuum mechanics because it is customary to measure all extensive variables per gram rather than per mole) ( Mase, 1970).

Assume air is a diatomic ideal gas ( that is, $\gamma$ = 1.4 ).

so that:

$$P(\rho, s) = \text{const. } \rho^{\gamma} e^{s/c_v}$$

Then,

$$\left(\frac{\partial P}{\partial \rho}\right)_S = \gamma \rho^{\gamma-1} \cdot \text{const} \cdot e^{S/c_v}$$

$$= \frac{\gamma P}{\rho} = \gamma PC$$

thus:

$$C^2 = \gamma P_o \tau_o = \gamma \overline{R} \theta_o$$

$$C^2 = \frac{\gamma R \theta_o}{M}$$

where R is the gas constant, M is the molecular weight.

given that $\frac{R}{M} = \frac{8.32 \times 10^7}{29}$,

and $\theta_o = 273^0 K$

thus $C = 331.14$ m/s

which is in good agreement with experimental value of C = 330 m/s.

## CONCLUSION

Sound propagation was investigated in a non-viscous fluid that was slightly disturbed. A perturbation analysis has been applied to the basic equations of continuum mechanics and thermodynamics to derive an expression for the velocity of wave propagation in a non-viscous fluid. The velocity of the sound wave for air at $0^0$C in the fluid from the theoretical formulation was 331.14 m/sec. This is approximately equal to the experimental value of 330 m/sec.


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