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ABSTRACT

In this study, effect of thermal radiation on the entropy generation rate of a hydromagnetic incompressible viscous flow through porous channel has been studied. The governing equations are formulated, non-dimensionalized and solved by Adomian decomposition and Differential Transform methods. The obtained velocity and temperature profiles are used to compute the entropy generation rate and Bejan number. The influence of various flow parameters on the velocity, temperature, entropy generation rate and Bejan number are discussed graphically.

(Keywords: thermal radiation, entropy generation, MHD, Bejan number, Adomian decomposition method, differential transform method)

INTRODUCTION

Radiative effects are of significant importance in many scientific and environmental processes like evaporation from large open reservoirs, heating and cooling of chambers, solar power technology, astrophysical flows, etc. They also play important roles in manufacturing industries in areas such as design of reliable equipment, nuclear power plants, gas turbines, and various propulsion devices for air craft, satellite, space vehicles, etc.

Radiative effects on fluid flow have been studied by numerous researchers by adopting Roseland or various radiative flux models like Raptis and Perdikis, Hall, Hakiem, Bakier, Rao, etc. Prominent among the researchers are Adesanya [1] who investigated the steady magnetohydrodynamic visco-elastic heat generating/absorbing slip flow through a porous medium with radiation effect using Rosseland approximation.

Das et al. [2] in studies on radiation effects on unsteady free convection flow past a vertical plate with Newtonian heating, they submitted that increase in radiation parameter decreases the fluid temperature.

Dada et al. [3] discussed dissipation, MHD and radiation effects on an unsteady convective heat and mass transfer in a Darcy-Forcheimer porous medium. They concluded that thermal radiation reduces the rate of energy transport to the fluid.

Vyas et al. [4] investigated the entropy regime for radiative MHD Couette flow inside a channel with naturally permeable base, showing that entropy generation increases with increased radiation parameter. For other interesting researches on radiation effects on various fluid flow, [see refs. 5-11]

Researchers’ interest have been drawn to MHD flow of electrically conducting viscous incompressible fluids due to its numerous industrial applications such as geothermal reservoirs, nuclear reactor, marine propulsion, electronic packaging, microelectronic device operations, textile industry, polymer technology, metallurgy, crude oil purification, and the cooling of reactors. Adesanya et al. [12] investigated the thermodynamics analysis of hydromagnetic third grade fluid flow through a channel filled with porous medium and discovered that an increase in magnetic field parameter decreases the fluid velocity while it increases the fluid temperature.

Again, Adesanya et al [13] presented the study on thermodynamic analysis for a third grade fluid through a vertical channel with internal heat generation, among the submissions is that increase in the Grashof number depletes the exergy level of the thermal system. In addition, heat transfer dominates the channel with increase in Grashof number, Alam et al. [14]
studied the effectiveness of viscous dissipation and Joule heating on steady magneto-hydrodynamic heat and mass transfer flow over an inclined radiative isothermal permeable surface in the presence of thermophoresis. They concluded that magnetic field parameter decreases the local Nusselt number due to the important Joule heating effect.

Turkyilmazohlu [15] presented the thermal radiation effects on the time-dependent MHD permeable flow having variable viscosity and submitted that magnetic field has increasing effects on temperature profiles while it decreases the radial and azimuthal skin friction values.

The optimal performance of most engineering flow processes and thermal systems has been a major concern due to the diminishing effect of available energy as a result of entropy generation. Therefore determination of the factors responsible for entropy generation is of great interest to engineers and mathematicians.

One of the methods being used to predict the performance of the engineering processes is the second law of thermodynamics. The pioneer of this method was Bejan [16, 17, 18]. Since then researchers have identified various reasons such mass transfers, fluid friction, magnetic field etc. as the cause of entropy generation. Therefore determination of the factors responsible for entropy generation is of great interest to engineers and mathematicians.

ADM is applied by splitting the given equation into linear and non-linear parts, inverting the highest-order derivative operator in the linear operator on both sides. The initial/boundary conditions together with the source term are identified as the zeroth component while the nonlinear term is decomposed as Adomian polynomials and the successive terms as series solution by recurrent relation using Adomian polynomials. The convergence of the results from both methods can be improved upon by increasing the number of iterates.

The objective of this study is to provide analysis of the effect of thermal radiation on the entropy generation rate of a hydromagnetic incompressible viscous flow through porous channel. The work is organized as follows: we present the formulation and non-dimensionalization of the problem; the solution to the boundary value problems; the results; and the conclusions are set out at the end of the paper.

**MODEL FORMULATION**

We assume the following:

(i) The flow is steady and incompressible.
(ii) The fluid is viscous and flow through porous channel.

(iii) A uniform magnetic field is applied in the presence of thermal radiation

(iv) Hall effect and induced magnetic field (because magnetic Reynolds number is very small for most fluid used in industrial applications) are neglected.

(v) Roseland approximation is applicable

Under these assumptions the governing equations are written as (Adesanya [1], Das et al. [22])

\[ \rho V_c \frac{du'}{dy'} = -\frac{dp}{dx} + \mu \frac{d^2u'}{dy'^2} + \frac{\sigma^e B^2 u'^2}{\rho} \]

\[ u(0) = 0 = u(h) \]

\[ \rho c_p \frac{dT'}{dy'} = k \frac{d^2T'}{dy'^2} + \mu \left( \frac{du'}{dy'} \right)^2 + \frac{\sigma^e B^2 u'^2}{\rho c_p} \]

\[ k \frac{dT'}{dy'}(0) = -\gamma_1(T_f - T') \]

and

\[ k \frac{dT'}{dy'}(h) = -\gamma_2(T' - T_c) \]

where \( u' \) is the axial velocity, \( \mu \) is the dynamic viscosity, \( p \) is the fluid pressure, \( h \) is the channel width, \( \rho \) is the fluid density, \( T' \) is the fluid temperature, \( T_0 \) is the initial fluid temperature, \( T_f \) is the final fluid temperature, \( k \) is the thermal conductivity of the fluid, \( c_p \) is the specific heat at constant pressure, \( v_0 \) is the constant velocity of fluid suction/injection, \( \sigma^e \) is the electrical conductivity of the fluid, \( \gamma_1,2 \) are the heat transfer coefficients, \( B_0 \) is the uniform transverse magnetic field, and \( q_r \) is the radiative heat flux. We invoke the Roseland approximation for radiation and have:

\[ q_r = \frac{4\sigma}{3k^*} \frac{dT'^4}{dy'} \]

where \( \sigma \) is the Stefan-Boltzman constant and \( k^* \) is the mean absorption coefficient for thermal radiation. Following Raptis and Perikis [30] we can express temperature functions in equation (5) as a linear function of temperature. Expanding \( T'^4 \) in a Taylor series about \( T_0 \) and neglecting second and higher order terms, we obtain:

\[ T'^4 \approx 4T_0^3T - 3T_0^3 \]

Substituting Equations (5) and (6) in (3), we get:

\[ \rho c_p \frac{dT'}{dy'} = k \frac{d^2T'}{dy'^2} + \mu \left( \frac{du'}{dy'} \right)^2 + \frac{\sigma^e B^2 u'^2}{\rho c_p} + \frac{16\sigma T_0^3}{3k^*} \frac{d^2T}{dy'^2} \]

Introducing the following non-dimensional quantities:

\[ y = \frac{y'}{h}, u = \frac{u'}{v_o}, \theta = \frac{T' - T_s}{T_f - T_s}, \]

\[ s = \frac{v_o h}{v}, G = -\frac{h^2}{\mu v_o} \frac{dp}{dx}, Pr = \frac{\nu p c_p}{k}, \]

\[ Br = \frac{\mu v_o}{k(T_f - T_c)}, B_i = \frac{\gamma_i h}{k}, \]

\[ B_i = \frac{\gamma_2 h}{k}, Ns = \frac{T_s^2 h^2 E_g}{k(T_f - T_c)^2}, \]

\[ \Omega = \frac{T_f - T_s}{T_c}, H^2 = \frac{\sigma B^2 h^2}{\rho}, R = \frac{k^*}{4\sigma T_0^3} \]

Using Equation (9) in Equations (1-8), we have the following boundary value problems:
\[ s \frac{du}{dy} = G + \frac{d^2 u}{dy^2} - H^2 u \quad u(0) = 0 = u(1) \]  
(9)

\[ \left(1 + \frac{4}{3} R \right) \frac{d^2 \theta}{dy^2} = sp \frac{d \theta}{dy} - Br \left( \frac{du}{dy} \right)^2 + BrH^2 u^2 \]  
(10)

\[ \frac{d \theta}{dy} = Bi_s (\theta - 1) \quad \text{on} \ y = 0 \quad \text{and} \quad \frac{d \theta}{dy} = -Bi_s \theta \quad \text{on} \ y = 1 \]  
(11)

where \( u \) is the dimensionless velocity, \( s \) is the suction/injection parameter, \( \theta \) is the dimensionless temperature, \( Pr \) is the Prandtl number, \( Br \) is the Brinkman number, \( \Omega \) is the parameter that measures the temperature difference between the two heat reservoirs, \( H \) is the magnetic field parameter, \( Be \) is the Bejan number, \( Bi_{s,1,2} \) are Biot numbers, and \( G \) is the axial pressure gradient, \( R \) is the thermal radiation parameter.

**METHOD OF SOLUTION**

**Solution Via Adomian Decomposition Method:** Writing Equation 9 as:

\[ u(y) = Dy + \int_0^y \int_0^y \left( -G + s \frac{du}{dy} + H^2 u \right) dy \]  
(12)

The zeroth component is identified as:

\[ u_0(y) = Dy - \int_0^y G dy \]  
(13)

while the remaining terms constitute the recursive relation written:

\[ u_{n+1}(y) = \int_0^y s \frac{du}{dy} + H^2 u dy \]  
(14)

Also Equation (10) is of the form:

\[ \theta(y) = E + Fy + \left( \frac{3}{3 + 4R} \right) \int_0^y \int_0^y \left[ s \frac{d \theta}{dy} - \frac{3}{Br} \left( \frac{du}{dy} \right)^2 + BrH^2 u^2 \right] dy \]  
(15)

The zeroth component is:

\[ \theta_0(y) = E + Fy \]  
(16)

and the recursive relation is given as:

\[ \theta_{n+1}(y) = \left( \frac{3}{3 + 4R} \right) \int_0^y \int_0^y \left[ s \frac{d \theta_n}{dy} - \frac{3}{Br} \left( \frac{du_n}{dy} \right)^2 + BrH^2 u^2 \right] dy \]  
(17)

**Solution Via Differential Transform**

\[ U(k + 2) = \frac{1}{(k + 2)!} \left[ s(k + 1)U(k + 1) + H^2 u(k) - G \delta(k) \right] \]  
(18)

\[ U(0) = 0, \quad U(1) = D \]

\[ \theta(k + 2) = \frac{1}{(k + 2)!} \left( \frac{3}{3 + 4R} \right) \left( s \frac{d \theta(k + 1)}{dy} - \right) \]

\[ Br \sum_{r=0}^{k} (r + 1)(k - r + 1)U(r + 1)U(k - r + 1) + H^2 \sum_{r=0}^{k} U(r)U(k - r) \]  
(19)

\[ \theta(0) = E, \quad \theta(1) = Bi_s (\theta - 1) \]  
(20)
The exact solution of Equation (9) is given as:

\[
\begin{aligned}
u(y) &= \left[ \frac{1}{2} - \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right) y \right] e^{\frac{1}{2} - \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right) - 1} \\
&= \frac{1}{2} \left[ 1 + \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right) y \right] e^{\frac{1}{2} - \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right)} \\
&= \frac{1}{2} \left[ 1 + \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right) y \right] e^{\frac{1}{2} - \frac{1}{2} \left( \frac{\sqrt{4 + H}}{H} \right)} + \frac{G}{H} \\
\end{aligned}
\]

(21)

**ENTROPY GENERATION**

Entropy generation analysis is for the prediction of the performance of the engineering processes. The method for entropy generation analysis is the second law of thermodynamics. The local entropy generation in Equation (23) shows the contributions of four sources. The first, second, third and fourth terms are local entropy generation due to heat transfer, radiation, fluid friction and magnetic field, respectively.

\[
E_G = \frac{k}{T^2} \left( \frac{dT^*}{dy^*} \right)^2 + \frac{16\sigma T_0^3}{3k} \left( \frac{d^2T}{dy^*} \right)^2 + \\
\mu \left( \frac{du^*}{dy^*} \right)^2 + \frac{\sigma B_s^2 u^2}{T_s}
\]

(23)

Using Equation (8) in (23) we obtain the dimensionless entropy number as:

\[
Ns = \left[ 1 + \frac{4}{3} R \left( \frac{d\theta}{dy} \right)^2 \right] + \\
Br \frac{\left( \frac{du}{dy} \right)^2}{\Omega} + H^2 u^2
\]

(24)

The Expression in (23) gives the spatial distribution of entropy generation but does not identify the contribution of each of the terms to total entropy generation. According to Bejan, \( N_s \) gives a summation of heat transfer irreversibility denoted by \( N_1 \) as irreversibility due to heat transfer while \( N_2 \) denotes irreversibility due to fluid friction with magnetic field.

\[
N_1 = \left[ 1 + \frac{4}{3} R \left( \frac{d\theta}{dy} \right)^2 \right], \\
N_2 = \frac{Br}{\Omega} \left( \frac{du}{dy} \right)^2 + H^2 u^2
\]

(25)

The Bejan number \( Be = 0 \) is the limit at which irreversibility due to viscous dissipation dominates while \( Be = 1 \) is the limit at which irreversibility due to heat transfer dominates the flow of fluid. \( Be = 0.5 \) is when heat transfer and fluid friction contribute equally to entropy generation. The Bejan number can be written as where \( \Phi \) is the irreversibility ratio.

\[
Be = \frac{N_1}{N_s} = \frac{1}{1 + \Phi}, \quad \Phi = \frac{N_2}{N_1}
\]

(26)

**RESULTS AND DISCUSSION**

In this study, effect of thermal radiation on the entropy generation rate of a viscous incompressible flow through porous channel has been solved using the Adomian decomposition method and differential transform method. The expressions for the velocity and temperature have been used to compute the entropy generation. The results showing the effects of some pertinent parameters on the velocity, temperature, entropy generation and Bejan number are presented and discussed graphically in Figures 1-9.

**Effect of Parameters Variation on Velocity and Temperature Profiles**

We present the influence of parameter variation on velocity and temperature of fluid in the following Figures. In Figure 1 the effect of magnetic field parameter on velocity is depicted, the plot reveals that increase in magnetic field parameter reduces the velocity. This is expected because of the opposing force exerted by magnetic field on the fluid velocity.

Figure 2 depicts the effect of suction/injection on velocity profile. It is found that fluid velocity decreases at the injection wall while it increases slightly at the suction wall.
Figure 3 illustrates the influence of variation in thermal radiation parameter on fluid temperature. The plot shows that fluid temperature reduces due to increase in radiative parameter. The reason is that increase in the radiative parameter $R$ increases the absorptivity rate $k$ thereby lowering the fluid temperature.

Furthermore Figure 4 depicts the influence of magnetic field parameter on fluid temperature. It is shown that increase in magnetic field parameter $H^2$ increases the fluid temperature. This is physically true due to additional source of heat from Ohmic heating.

In Figure 5, the effect of suction/injection on fluid temperature is displayed. Expectedly, we observed an increase in temperature as hot fluid is being injected into the channel increases.
Effect of Parameters Variation on Entropy Generation Rate

We present the influence of parameters variation on entropy generation in the following figures.

In Figure 6 the effect of thermal radiation on entropy generation is displayed. We observe that entropy generation increases across the channel as radiative parameter becomes higher. The reason can be attributed to the rise in emission of radiation.

Figure 7 illustrates the effect of suction/injection on entropy generation rate. There is a reduction in entropy generation as suction/injection parameter increases in values.

![Figure 6: Entropy Generation Rate for Thermal Radiation Parameter.](image1)

![Figure 7: Entropy Generation Rate for Suction/Injection.](image2)

Effect of Parameters Variation on Bejan Number

In Figure 8 the effect of thermal radiation parameter on Bejan number is depicted. It is noticed that increase in radiative parameter increases Bejan number which indicates that heat transfer irreversibility dominates entropy generation.

Figure 9 displays the effect of suction/injection on Bejan number. The graph shows that entropy generation decreases at the injection wall and rises at the suction wall. The implication of this is that viscous dissipation is dominant at the injection channel while heat transfer irreversibility dominates at the suction wall.

![Figure 8: Effect of Thermal Radiation Parameter on Bejan Number.](image3)

![Figure 9: Effect of Suction/Injection on Bejan Number.](image4)
Table 1: Computation Showing Convergence of Solution when 

\[ G = s = H = 1. \]

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CONCLUSION

This study investigates the effect of thermal radiation on the entropy generation rate of a hydromagnetic incompressible viscous flow through porous channel. The solution of the governing momentum and energy equations obtained by Adomian decomposition and differential transform methods are used to compute the entropy generation rate. The influences of various physical parameters are discussed graphically. The main conclusions are stated below:

(i) Radiation parameter reduces fluid temperature while the temperature rises with increase in Hartman number, Brinkman number, Prandtl number and suction/injection.

(ii) Entropy generation is higher at the upper wall of the channel.

(iii) Heat transfer irreversibility is the dominant contributor to entropy generation at the upper wall.

REFERENCES


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