Analytical Velocity Model for Depth Conversion in the Subsurface Facies of Agbada Formation in the Niger Delta, Nigeria.

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ABSTRACT

An analytical velocity model that better approximates the effects of gradually increasing velocities with depth due to compaction in the facies of Agbada Formation in the Niger Delta is established in this study. Most existing models assume that lithology is sharply discontinuous, discrete, and that velocity is laterally or vertically constant within a particular medium, all of which are mostly improbable. An instantaneous linear velocity model which finds proper application in thin clastic sediments is defined by two analytic parameters: (1) top-interface velocity, \( V_0 \) and (2) velocity gradient or compaction factor, \( k \). The analyses of data from the Central-Niger Delta, Nigeria show that \( V_0 \) and \( k \) have equivalent values of 1656 ms\(^{-1}\) and 0.44 s\(^{-1}\) respectively. The derived velocity function: \( V(z) = 1656 + 0.44kz \), provides a basis for conveniently predicting velocities at various depth intervals useful in time-depth conversion of the subsurface clastic-sediments of the Niger Delta. The derived \( V_0 \) can also be used as a static correction parameter in this part of the Niger Delta.

(Keywords: top interface velocity, compaction factor, depth conversion, facies of Agbada Formation, Niger Delta)

INTRODUCTION

Velocity modelling for depth conversion has been faced with many challenges especially with the complexities involved in applying the correct velocity function to the appropriate geology for proper sub-surface investigations.

In areas where there are dramatic changes of velocity, such as thick carbonate or evaporate units alternating with thick elastic units (as those found in the southern North Sea basin), complex structures, tectonic inversions or lateral lithology change, the layer cake model which treats each lithologic unit separately and defines each unit by a different mathematical function may be appropriate for depth conversion. But in areas of uniform lithology which have not undergone tectonic inversion or severe structural deformation, depth conversion can often be performed according to some effective mathematical functions (instantaneous velocity functions) that respect the geology of the area under consideration.

In many clastic sedimentary basins of the world, velocity gradually increases with depth. Lithology like velocity in these areas actually changes gradually with depth of burial rather than in discrete steps at boundaries. The rate of change is erratic and faster in the near-surface and decreases with a regular trend in the deeper subsurface due of the effect of compaction. This is the basis of the instantaneous velocity depth model which has enjoyed some degree of popularity in the industry.

The simplicity and wide application of this model (Slotnick, 1936; Al-Chalabi, 1997b and 2001; Smallwood, 2002; Ravve and Koren, 2006; among others) show that it tends to work well in clastic sediments, hence its choice for this study in the Niger Delta which compares with the Gulf Coast of United States (Morley and Guerin, 1996 and Wu and Bally, 2000) where it has been widely applied. This study essentially derives top interface velocity, \( V_0 \), compaction factor, \( k \) and the velocity function useful for predicting lateral and vertical velocity structure of the deep subsurface and converting time-to-depth section in the Niger Delta. Compaction factor, \( k \) value is generally between 0.3 and 1.3 per sec (Telford et al., 1976). In the Gulf of Mexico, the vertical velocity gradient, \( k_z \) is commonly around 0.6 s\(^{-1}\).
(Sheriff, 1991), and in Western Canada, $k_z$ is between $0.25 - 1.0 \text{ s}^{-1}$ (Jain, 1987). According to Xu et al. (1993), typical values of vertical velocity gradient lie in the range $0.6 - 1.0 \text{ s}^{-1}$.

**SUBSURFACE STRUCTURE OF THE NIGER DELTA**

As in many deltaic areas, it is extremely difficult to define a satisfactory stratigraphic nomenclature (Doust and Omatsola, 1990). The interdigitation of a small number of lithofacies makes it impossible to define units and boundaries of sufficient integrity to constitute discrete and sharply discontinuous formations in a formal sense. However, three formation names are in wide-spread use (Short and Stauble, 1967 and Avbovbo, 1978), corresponding to the portions of the tripartite sequence (Figure 1).

The first is known as the Marine Shales. This lithofacie is composed of shales, clays and silts at the base of the known delta sequence. They contain a few streaks of sand, possibly of turbiditic origin, and were deposited in holomarine (delta-front to deeper marine) environments. The thickness of this sequence is not known but may reach 7000m in the central part of the delta. Marine shales form the base of the sequence in each depobelt and range from Paleocene to Holocene in age. They crop out offshore in diapirs along the continental slope, and onshore in the North-Eastern part of the delta, where they are known as the Imo Shale. Except on the basin flanks, no wells have fully penetrated this sequence. The marine shale sequence is typically over-pressured.

Overlying the Marine Shales is the Paralic Clastics, the facies of Agbada Formation (Fm). This forms the hydrocarbon-prospective sequence in the Niger Delta. It is represented by an alternation of sands, silts and clays in various proportions and thicknesses, representing cyclic sequences of offlap units. These facies are the truly deltaic portions of the sequence and were deposited in a number of delta-front, delta-topset, and are of fluvio-deltaic environments.

![Figure 1: The Different Lithofacies in the Niger Delta (based on Merki, 1972 and Weber and Daukoru, 1975, and modified by Whiteman, 1982).](image-url)
The alternation of the fine and coarse clastics provides multiple reservoir-seal couplets. As with the marine shales, the facies of Agbada Fm. is present in all depobelts, and ranges in age from Eocene to Pleistocene. Most exploration wells in the Niger Delta have bottomed in this lithofacies, which reaches a maximum thickness of more than 3000m. This formation has its surface outcrops as Ogwashi, Asaba and Ameki Formations. Overlying the facies of paralic Agbada Formation is the facies of the continental Benin formation which is the overlying Continental Sand.

The shallowest part of this sequence is composed almost entirely of non-marine sands. It was deposited in alluvial or upper coastal plain environments following a southward shift of deltaic deposition into a new depobel. The oldest continental sands are probably Oligocene, although they lack fauna and are impossible to date directly. Offshore they become thinner and disappear near the shelf edge. The present outcrops of this formation could be seen around Owerri, Benin and Onitsha.

FIELD LOCATION AND DATA

The study area is the Agbada field located in OML 17 approximately 16 km northeast of Port Harcourt. The Survey is situated between Latitude 4°55' and 5°10' north and between longitude 6°50' and 7°10' east in the Central Niger Delta of Nigeria (Figure 2).

A 3-D Seismic Reflection Survey was conducted from which various Time-Offset data were extracted. The prospect consisted of a regular grid configuration of 73 north-by-south running receiver lines increasing by 5 and 60 west-by-east source lines increasing by 8 (Figure 3). Receiver spread of 480 channels divided into six separate lines of 80 stations each was used throughout the program. Receiver and source lines were spaced at 250 and 400 m respectively. Geophones and source pegs were evenly spaced at 50 m. A 15-fold, non-symmetric split-spread geophones and shots were covered. The explosive energy source comprising 0.2 kg dynamite buried in 5 or 10 pattern holes each 3.0 or 6.0 m deep was used. On the other hand, checkshot velocity surveys (Figure 4) were also conducted in wells in the Agbada Field.

MODELING APPROACH

Overview of Analytical Velocity Models

Analytical velocity models assume that velocity varies in a systematically continuous manner with depth. A comprehensive description of these models that respect the geology of different areas had been given by various researchers as shown in Table 1 and Figure 5. Details of a variety of other given by Kaufman, 1953.

The simplest of the analytical velocity models chosen for this study is the instantaneous linear velocity function which tends to work well in thin clastic sections and is based on Equation 1 (Slotnick, 1936):

\[ V(z) = V_0 + kz \]  \hspace{1cm} (1)

and \( V(z) = \frac{dz}{dt} \) (Marsden, 1992) \hspace{1cm} (2)

Equation (1) assumes that instantaneous velocity, \( V(z) \) in m/s changes linearly with lithologic depth, \( z \) in m. It is defined by two analytical parameters: \( V_0 \) (top interface velocity in m/s) and \( k \) (the rate of velocity increases with depth or the compaction factor in s\(^{-1}\)) for the given medium.
Figure 3: Agbada 3-D Program Map showing Receiver and Source Line, (SPDC FDP Map).
Figure 4: Schematic Representation of a Typical Checkshot Velocity Survey on Land (this study).

Equation (2) states that the instantaneous velocity is the rate of change of depth, \( dz \) with time, \( dt \).

**MODEL PARAMETERIZATION**

\( V_0 \) and \( k \) have been described as parameters of convenience to provide a simple description of how the instantaneous velocity varies with depth (Al-Chalabi 1997a). In deriving these parameters, it was initially ascertained, by the shape of the various curves, that the data in this part of the Niger Delta fits the theory of linear increase of velocity with depth.

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**MODEL PARAMETERIZATION**

**T-X Curves:**  T-X data from 3-D Seismic were plotted and a 'smooth' curve of the anti-hyperbolic sine type was fitted according to Equation (3) (Slotnick, 1936) and shown in Figure 6a:

\[
T = \frac{2}{k} \sinh^{-1} \left( \frac{kX}{2V_0} \right)
\]

Where:
- \( T \) = Time of arrival in seconds
- \( X \) = Distance between shotpoint and receiver in metres

**T-Z Curves:**  T-Z data from Checkshots were plotted and a 'smooth' curve of the anti-hyperbolic cosine type was fitted according to Equation (4) (Slotnick, 1936) and shown in Figure 6b:

\[
t = \left( \frac{1}{k} \right) \cosh^{-1} \left( \frac{1 + k^2 (d^2 + z^2)}{2V_0} \right)
\]

Where:
- \( d \) = Distance of shotpoint from wellbore centre in metre.
- \( z \) = Hydrophone offset in metre
- \( t \) = One-way arrival time in seconds

The 3-D seismic and checkshot profiles (Figures 6a and 6b) are observed to be substantially alike; this implies that the subsurface sections are laterally and horizontally 'normal', that is, without steep dips or complex structures satisfying the conditions for the application of the theory of linear increase of instantaneous velocity with depth.

**V₀ – K Parameters**

The 3-D Seismic data (high sampling density) provided the \( V_0 \) map in the lateral direction. This is determined by solving Equation (5) (Slotnick, 1936) analytically:

\[
\sinh \left( \frac{kT}{2} \right) = \frac{kX}{2V_0}
\]

The Checkshots data (high certainty and low sampling density) provided the \( k \)-gradient in the vertical direction. This is determined by solving Equation (6) (Slotnick, 1936) analytically:
Table 1: Summary of Analytic Velocity Models.

<table>
<thead>
<tr>
<th>MODEL EQUATION</th>
<th>DEFINITION OF PARAMETERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V(z) = V_0 + k_z )</td>
<td>( V(z) ) is the top-interface instantaneous velocity, ( k ) is a constant vertical gradient, ( z ) is lithologic depth</td>
</tr>
<tr>
<td>( V(z) = V_0 \exp(k_z / V_0) )</td>
<td>( V(z) ) is the top-interface instantaneous velocity, ( k_0 ) is the gradient at the top interface, ( z ) is lithologic depth</td>
</tr>
<tr>
<td>( V(z) = A\sqrt{\pi z} \quad n = 6 ) or ( V(z) = V_a \cdot \sqrt{\pi} \cdot n \cdot k_z / V_a )</td>
<td>( V(z) ) is the top-interface instantaneous velocity, ( V_a ) is the top-interface velocity, ( k_0 ) the top-interface gradient, ( n ) the root index</td>
</tr>
<tr>
<td>( V(z) = \sqrt{V_a^2 + 2V_a k_z z} )</td>
<td>Has the same parameters as Faust (1951 &amp; 1953) but with ( n = 2 )</td>
</tr>
<tr>
<td>( V(z) = \frac{V_1^2}{(V_a - k_z z)} )</td>
<td>( S ) is slowness (reciprocal of velocity), ( A ) is a constant, ( V(z) ) is the top-interface instantaneous velocity, ( V_a ) the top-interface velocity, ( k_0 ) the top-interface gradient and ( S_0 ) slowness at the top interface</td>
</tr>
<tr>
<td>( V(z) = V_1 + k_z z - \frac{k_0^2}{2} )</td>
<td>( V_a ) is the top-interface velocity, ( k_0 ) the top-interface gradient, ( z ) is lithologic depth</td>
</tr>
<tr>
<td>( V_{a,2}(z) = V_{a,1} + \Delta V_{a,1} \cdot \left[ 1 - \exp\left( -\frac{k_0^2}{\Delta V_{a,1}} \right) \right] ) and ( V_{a,1} = V_{a,0} + \Delta V_{a,0} )</td>
<td>( V_{a,2}(z) ) is instantaneous velocity function vs. relative depth, ( V_{a,1} ) is instantaneous velocity at top interface of layer, ( k_0 ) the vertical velocity gradient at the top interface, ( V_{a,0} ) is instantaneous velocity range, ( V_{a,\infty} ) is the asymptotic velocity at the infinite depth</td>
</tr>
</tbody>
</table>

\( V_0 = \frac{k_0}{2} \sqrt{1 + 2 \left( \frac{d^2}{z^2} \right) + 1} \coskh - 1 \) \( r \) \( (6) \)

These values are geostatistically combined using the kriging algorithm in SURFER 8.0™ software to predict unknown values at other locations with poor or no data in this field.

RESULTS

Computations of the model parameters applying the linear increase of velocity with depth yielded good \( k \) and \( V_0 \) results for the facies of Agbada Fm. at the Central Niger Delta.

Velocity Gradient, k Values

The equivalent value of 0.44 s\(^{-1}\) obtained for \( k \) defines the rate at which velocity increases with depth in this field. A contour obtained by kriging the various values of \( k \) from the checkshots data in the different wells within this field is shown in Figure 7. Though \( k \) is observed to have slight lateral variation increasing from east to west of the D5200 horizon in this prospect, it is constant in the vertical direction. This slight lateral variation gives an indication of the sediments transport and deposition sequence within this field. Vertical velocity changes within this field depend on \( k \), compaction factor of these clastic sediments. \( V_0 \) from the 3-D Seismic data was now computed based on this equivalent value of \( k \).
Top Interface Velocity, $V_0$

An equivalent value of 1655.693 ms$^{-1}$ for $V_0$ was determined for this field. Fig. 8 is a contour of the various $V_0$ values geostatistically obtained by Kriging using SURFER 8.0 software.

The west-central part of the prospect was observed to have the lowest value of 1565 ms$^{-1}$ while in the north-eastern part of the prospect; $V_0$ was as high as 1705 ms$^{-1}$ indicating that $V_0$ is not constant on a horizon but varies in the lateral direction as against the previous assumption that velocity is constant in the medium within this field.

$V_0$-K DEPTH CONVERSION

The use of an instantaneous velocity function to convert seismic times to depth gives excellent results in sedimentary basins with good well control (Smallwood, 2002). 3-D seismic one-way travelttime, $t$ obtained on the D5200 horizon at 60 well positions in the Agbada field of the Niger Delta basin were converted to depth using the $V_0$–k parameterization. The formula derived from Equations (1) and (2) which relates depth, $z$ and the one-way seismic travelttime, $t$ used for this depth conversion is given (Marsden, 1992) as:
Figure 6a: Model T-X Curve for the Agbada Field 3-D Seismic.

Figure 6b: Model T-Z Curve for Agbada Checkshot Well – 43 (This study).
Figure 7: k-Gradient Contour Map for the Agbada Field (This study).

Figure 8: V₀ Contour Map for the Agbada Field.
\[ z = \frac{V_0}{k} \left( \kappa_r - 1 \right) \]  \hspace{1cm} (7)

Figure 9 is an isopach map of the converted depths obtained by kriging for the two horizons across the field.

**VELOCITY GRADIENT FUNCTION FOR THE AGBADA FIELD**

These equivalent values of \( k = 0.44 \text{ s}^{-1} \) and \( V_0 = 1655.693 \text{ ms}^{-1} \) were used to build, based on Equation (1), the empirical model which satisfies the condition that velocity increases linearly with depth due to compaction in these facies. The velocity function is derived as:

\[ V(z) = 1.656 + 0.44z \]  \hspace{1cm} (8)

Where: \( V_0 \) is now in km and \( k \) in s\(^{-1}\).

This relationship represents the velocity variation below the weathered low velocity layer of the facies of Agbada Fm. in the Niger Delta sedimentary basin made up of thin clastic materials consisting of broken fragments of rocks which include siliciclastic rocks such as conglomerates, sandstones, siltstone and shale.

**VELOCITY-DEPTH MODEL CURVE FOR THE DEEP SUBSURFACE**

A plot of \( V(z) \) values derived by Equation (8) against the depth-converted, \( z \) values by Equation (7) gives the model curve shown in Figure 10.

*Figure 9: Isopach Map for the D5200 Horizon of the Agbada Field (This study).*
The curve shows a linear relationship with a gentle slope, \( k \) (velocity gradient). An extrapolation of the curve to the \( V(z) \)-axis gives the intercept, \( V_0 \) in the velocity function in Equation (8).

The trend suggests that velocity increases linearly with depth in the subsurface below the weathered (Low Velocity) zone of the Niger Delta. Within this zone, a considerable level of compaction and consolidation has been achieved by the clastic materials.

**CONCLUSION AND RECOMMENDATION**

The determination of best-fit \( V_0 \) and \( k \) parameters by the geostatistical integration of seismic and well (checkshot) data has allowed the development of an effective velocity model for conveniently carrying out depth conversion over a wide range of subsurface depths in the Agbada facies of the onshore Central Niger Delta. The important advantage of this instantaneous velocity-depth function is that depths to any horizon mapped within these clastic materials can be estimated with the same model parameters. In this region, with its relatively homogeneous, normally pressured siliciclastic fill, simple \( V_0 \)-\( k \) model for depth conversion as shown here gives good results, but care should be taken when applying such a model to other areas having complex geology.

This method provides a basis for better approximation of the depth structure of subsurface-clastic sediments using relatively small layer thickness to accurately estimate geologically important seismic events such as changes in facies, fractures, faults, and unconformities and identify structural closures for better hydrocarbon target. Recommendation is hereby made that software vendors working in the Niger Delta adopt this velocity model for
developing depth conversion algorithms suitable for application in the Niger Delta, particularly since there seems to be no record of its application in the Niger Delta, Nigeria.

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REFERENCES


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