Analysis of the Effect of Viscous Dissipation on the Temperature Profile of the Laminar Flow in a Channel Filled with Saturated Porous Media.

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ABSTRACT

The effect of viscous dissipation on the temperature profile of the laminar flow in a channel filled with saturated porous media is analyzed. The source of inspiration of this work could be found in Makinde et al. [4]. In this new paper, we include the viscous dissipation term which has an effect on the temperature profile of the fluid flow in the channel. The Brinkman model was employed. Velocity and temperature profiles are obtained for large Brinkman number using Pascal programming language and Maple software packages. Generally our result shows that the fluid temperature decreases with a decrease in porous media permeability (s increases) as the Brinkman number Br, increases.

(Keywords: laminar flow, porous medium, temperature profile, viscous dissipation)

NOMENCLATURE

We shall introduce the concept of Laminar flow in a channel filled with saturated porous media in this section. We give the definition of some parameters that feature prominently in this write-up and except where otherwise stated, these parameters will assume the definition given.

a	channel width
Br	Brinkman number
c_p	specific heat at constant pressure
Da	Darcy number
G	applied pressure gradient
k	fluid thermal conductivity
K	permeability
M	ratio of viscosities
Pe	Péclet number
Q	fluid flux rate
s	$\frac{1}{\sqrt{MDa}}$
Re	Reynolds number
T_o	wall temperature
T	absolute temperature
u	dimensionless fluid velocity as $s \to 0$
$\bar{u}$	fluid velocity
$\bar{x}$	axial coordinate
y	dimensionless transverse coordinate
$\bar{y}$	transverse coordinate

Greek Symbols

β	Frank-Kamenentskii parameter
μ	fluid viscosity
μ_e	effective viscosity in the Brinkman term
θ	dimensionless temperature
ρ	fluid density

Dimensionless Group

$x = \frac{\bar{x}}{P_e a}$	dimensionless axial coordinate
$y = \frac{\bar{y}}{a}$	dimensionless transverse coordinate
$u = \frac{\mu \bar{u}}{Ga^2}$	dimensionless velocity
$M = \frac{\mu_e}{\mu}$	ratio of viscosities
$Da = \frac{K}{a^2}$	dimensionless Darcy number

$Pe = \frac{\rho c_p G a^3}{k}$ dimensionless Péclet number

$Br = \frac{G^2 a^3}{q\mu}$ dimensionless Brinkman number

$\theta = \frac{k}{qa}(T - T_0)$ dimensionless temperature

INTRODUCTION

The importance of various studies concerning laminar flow in a channel filled with saturated porous media has attracted the interest of many researchers in recent times. Laminar flow configurations are generally observed in field of geothermal system, nuclear waste disposal, electronic cooling system, petroleum industries, solid matrix, heat exchanger, etc. (Makinde et al. [4]). The major problems in the engineering processes is the improvement in thermal systems and energy utilization during the convection in any fluid; this is because the thermal systems will provide better energy conservation, material processing and environmental effects see (Makinde [3]).

Thermoacoustic prime movers and heat pumps are other application areas of convection processes in porous media (Rott [7], Swift [9]) where the fluid-gap within stacks of a thermoacoustic engine and refrigerator are treated as porous media.

Most of the existing theories (of thermoacoustic engines and prime movers) consider a non-porous medium and very few of them use a single pore (of circular or square cross-section) to model thermoacoustic system Makinde et al. [4].

Other papers of interest can be found in Bader and Kambiz [1]. In their work, the boundary conditions for constant wall heat flux in the absence of local thermal equilibrium conditions are analyzed. Effects of variable porosity and thermal dispersion are also analyzed. The effects of pertinent parameters such as porosity, Darcy number, Reynolds number, inertia parameter, particle diameter and solid-to-fluid conductivity ratio, were discussed. Quantitative and qualitative interpretations of the results are utilized to investigate the prominent characteristics of the models under consideration. Limiting cases resulting convergence or divergence of the models are also considered. Results are presented in terms of the fluid, solid and total Nusselt numbers.

The hydromagnetic mixed convection flow of an incompressible viscous electrically conducting fluid and mass transfer over a vertical porous plate with constant heat flux embedded in a porous medium is investigated by Makinde [5]. Using the Boussinesq and boundary-layer approximations, the fluid equations for momentum, energy balance and concentration governing the problem are formulated. These equations are solved numerically by using the most effective Newton–Raphson shooting method along with fourth-order Runge–Kutta integration algorithm. It was found that for positive values of the buoyancy parameters, the skin friction increased with increasing values of both the Eckert number (Ec) and the magnetic field intensity parameter (M) and decreased with increasing values of both the Schmidt number (Sc) and the permeability parameter (K).

Sauoli and Aiboud-Sauoli [8] investigated second law analysis of laminar falling liquid film along an inclined heated plate. They consider the upper surface of the liquid film free and adiabatic and the lower wall is fixed with constant heat flux, their results show that entropy generation number transversely increases for all values of group parameter. Likewise they found that the irreversibility ration decreases in the transverse direction and increases as the group parameter increases. Their results also revealed the possibility of irreversibility ratio $\phi \geq 1$ for some group parameter values (i.e., the function irreversibility dominates over the heat transfer irreversibility).

In Makinde et al. [4], the entropy generation rate in a laminar flow through a channel filled with saturated porous media was investigated. The upper surface of the channel was adiabatic and the lower wall was assumed to have a constant heat flux. The Brinkman model was employed to obtained Velocity and temperature profiles are obtained for large Darcy number (Da) and used to obtain the entropy generation number and the irreversibility (i.e., $0 \leq \phi \leq 1$), and viscous dissipation has no effect on the entropy generation rate at the centre line of the channel. Meanwhile, several authors have also investigated the effect of temperature-dependent viscosity on the flow of the non-Newtonian fluids in a channel under various conditions, such as [2, 6, 10, 11].

The objective of this work is to formulate new problems on laminar flow in a channel filled with saturated porous media and to examine the effect of viscous dissipation on the temperature profile of the laminar flow in the channel filled with porous media.

MATHEMATICAL FORMULATION

We present in this work, the equations and boundary conditions in the mathematical models for the laminar flow in our work. We consider a coupled momentum and heat transfer by laminar flow for the steady state hydro-dynamically developed situation which have unidirectional flow in the x-direction between impermeable boundaries at $\bar{y}=0$ *and* $\bar{y}=a$ as in Figure 1. The channel is composed of a fixed lower heated wall with constant heat flux while the upper wall is fixed and adiabatic. Other physical properties of the fluid like viscosity and density are taken as constant. We follow closely and modify the models presented in Makinde et al [4] with additional term of viscous dissipation.

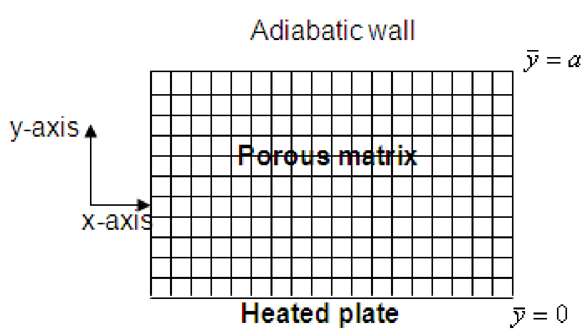

Figure 1: Problem Geometry.

In the following sections, the coupled non-linear steady-state momentum and energy balance equations, which govern the problem is obtained and subsequently, the resulting boundary-value problem is solved.

The Brinkman momentum equation is:

$$\mu_e \frac{d^2\bar{u}}{dy^2} - \frac{\mu}{K}\bar{u} + G = 0, \quad \bar{u}(0)=0, \quad \bar{u}(a)=0 \qquad \textbf{(1)}$$

Where μ_e is an effective viscosity, μ is the fluid viscosity, K is the permeability, and G is the applied pressure gradient.

The dimensionless form of Equation (1) is:

$$M\frac{d^2u}{dy^2} - \frac{u}{Da} + 1 = 0, \; u(0)=0, \; u(1) \qquad \textbf{(2)}$$

Together with the boundary conditions:

$$\text{u(y)=0 at y=0, u(y)=0 at y= 1} \qquad \textbf{(3)}$$

Method of Solution

Using the Maple software package to solve (2) with (3), we have:

$$u(y) = Da + d_1\sinh(ys) + d_2\cosh(ys) \qquad \textbf{(4)}$$

Where

$$s = \sqrt{\frac{1}{MDa}} = \frac{1}{\sqrt{MDa}}; \quad d_1 = Da\left(\frac{\cosh(s)-1}{\sinh(s)}\right); \quad d_2 = -Da \qquad \textbf{(5)}$$

It will be noted that , M and Da appear only in the combination of M times Da, hence, without loss of generality, we take M=1 in our analysis.

$$Da = \frac{1}{s^2}$$

Using the algebraic package (Taylor) in Maple, the Taylor expansion of (4) (for large Da) yields:

$$u(y) = \frac{y-y^2}{2} + \left(-\frac{y^4}{24} + \frac{y^3}{12} - \frac{y}{24}\right)s^2 + 0(s^4) \quad as \; s \to 0 \qquad \textbf{(6)}$$

The steady-state thermal energy equation for the problem is given as:

$$\rho c_p \bar{u}\frac{\partial T}{\partial \bar{x}} = k\frac{\partial^2 T}{\partial \bar{y}^2} + \mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}\right)^2 \quad \textbf{(7)}$$

With the following inlet and boundary conditions
Inlet condition:

$$T(0,\bar{y}) = T_0 \quad \textbf{(8)}$$

Constant heat flux at the lower wall,

$$\frac{\partial T}{\partial \bar{y}}(\bar{x},0) = -\frac{q}{k} \quad \textbf{(9)}$$

Adiabatic wall,

$$\frac{\partial T}{\partial \bar{y}}(\bar{x},a) = 0 \quad \textbf{(10)}$$

Where T is the absolute temperature at the inlet. The dimensionless energy equation is given as:

$$u\frac{\partial \theta}{\partial x} = \frac{\partial^2 \theta}{\partial y^2} + Br\left(\frac{\partial u}{\partial y}\right)^2 \quad \textbf{(11)}$$

$$\theta(0,1) = 0; \qquad \frac{\partial \theta}{\partial y}(x,0) = -1; \qquad \frac{\partial \theta}{\partial y}(x,1) = 0 \quad \textbf{(12)}$$

Now, solving (11) subject to (12), we employed the analytical method of separating the variables
Let,

$$\frac{\partial \theta}{\partial x} - u(y)^{-1}\frac{\partial^2 \theta}{\partial y^2} - Br\, u(y)^{-1}\left(\frac{\partial u}{\partial y}\right)^2 = 0 \quad \textbf{(13)}$$

Then,

$$\frac{\partial \phi_1}{\partial x} = d_3 \quad \textbf{(14)}$$

And,

$$\frac{\partial^2 \phi_2}{\partial y^2} = \left(\frac{y-y^2}{2} + \left(-\frac{y^4}{24} + \frac{y^3}{12} - \frac{y}{24}\right)s^2\right)d_3 - \frac{Br}{576}\left(-576 - 12y^2 + 144 + (-24 + 48y - 144y^2 + -384y^3 + 192y^4)s^2 + \left(1 - 12y^2 + 8y^3 + 36y^4 - 48y^5 + 16y^6\right)s^4\right) \quad \textbf{(15)}$$

Integrating (14) w.r.t x, we have:

$$\phi_1(x) = d_3\, x + d_4 \quad \textbf{(16)}$$

Also integrating (15) w.r.t. y, we have:

$$\phi_2(y) = Br\left(\left(\frac{y^2}{8} - \frac{y^3}{6} + \frac{y^4}{12}\right) + \left(-\frac{y^2}{48} + \frac{y^3}{72} + \frac{y^4}{48} - \frac{y^5}{30} - \frac{y^6}{90}\right)s^2\right)$$
$$+ Br\left(\frac{y^2}{1152} - \frac{y^4}{576} + \frac{y^5}{1440} + \frac{y^6}{480} - \frac{y^7}{504} + \frac{y^8}{2016}\right)s^4 \qquad \textbf{(17)}$$
$$+ d_3\left(\frac{y^3}{12} - \frac{y^4}{24} + \left(-\frac{y^3}{144} + \frac{y^5}{240} - \frac{y^6}{720}\right)s^2\right) + d_4 y + d_5$$
$$\therefore\ \theta(x,y) = \phi_1(x) + \phi_2(y)$$

i.e.

$$\theta(x,y) = d_3 x + d_4 + Br\left(\left(\frac{y^2}{8} - \frac{y^3}{6} + \frac{y^4}{12}\right) + \left(-\frac{y^2}{48} + \frac{y^3}{72} + \frac{y^4}{48} - \frac{y^5}{30} - \frac{y^6}{90}\right)s^2\right)$$
$$+ Br\left(\frac{y^2}{1152} - \frac{y^4}{576} + \frac{y^5}{1440} + \frac{y^6}{480} - \frac{y^7}{504} + \frac{y^8}{2016}\right)s^4 \qquad \textbf{(18)}$$
$$+ d_3\left(\frac{y^3}{12} - \frac{y^4}{24} + \left(-\frac{y^3}{144} + \frac{y^5}{240} - \frac{y^6}{720}\right)s^2\right) + d_4 y + d_5$$

But $\theta(0,1) = 0$

So (18) becomes,

$$\left(\frac{1}{24} - \frac{s^2}{240}\right)d_3 + \left(\frac{1}{20} - \frac{s^2}{120} + \frac{17s^4}{40320}\right)Br + 2d_4 + d_5 = 0 \qquad \textbf{(19)}$$

Also differentiating (18) w.r.t y and substituting the boundary conditions

$$\frac{\partial\theta}{\partial y}(x,0) = -1 \ and \ \frac{\partial\theta}{\partial y}(x,1) = 0 \quad respectively, \ we\,have \qquad \textbf{(20)}$$

$$d_4 = -1 \qquad \textbf{(21)}$$

and,

$$Br\left(\frac{1}{20} - \frac{s^2}{60} + \frac{17s^4}{20160}\right) + d_3\left(\frac{1}{12} - \frac{s^2}{120}\right) - 1 = 0 \qquad \textbf{(22)}$$

$$\therefore\ d_3 = \frac{1}{168}\left(\frac{(1680 - 336s^2 + 17s^4)Br - 20160}{s^2 - 10}\right) \qquad \textbf{(23)}$$

Putting (21) and (23) into (19), we have

$$d_5 = \frac{3}{2} \qquad \textbf{(24)}$$

So (18) becomes:

$$\theta(x,y)=\frac{1}{168}\left(\frac{(1680-336s^2+17s^4)Br-20160}{s^2-10}\right)\left(x+\frac{y^3}{12}-\frac{y^4}{24}+\left(-\frac{y^3}{144}+\frac{y^5}{240}-\frac{y^6}{720}\right)s^2\right)$$
$$+Br\left(\left(\frac{y^2}{8}-\frac{y^3}{6}+\frac{y^4}{12}\right)+\left(-\frac{y^2}{48}+\frac{y^3}{72}+\frac{y^4}{48}-\frac{y^5}{30}-\frac{y^6}{90}\right)s^2\right)$$
$$+Br\left(\frac{y^2}{1152}-\frac{y^4}{576}+\frac{y^5}{1440}+\frac{y^6}{480}-\frac{y^7}{504}+\frac{y^8}{2016}\right)s^4+\frac{1}{2}-y \qquad (25)$$

Now taking Br=0, we have,

$$\theta(x,y)=\frac{-120}{s^2-10}\left(x+\frac{y^3}{12}-\frac{y^4}{24}+\left(-\frac{y^3}{144}+\frac{y^5}{240}-\frac{y^6}{720}\right)s^2\right)+\frac{1}{2}-y$$

i.e

$$\theta(x,y)=\frac{-120x}{s^2-10}+\frac{30y^4-60y^3+s^2y^6-3s^2y^5+5s^2y^3+60y-6ys^2-30+3s^2}{6(s^2-10)} \qquad \textbf{(26)}$$

Here (26) is the result of Makinde et al [4] whereas (25) is our new result showing the effect of viscous dissipation on the flow.

RESULTS AND DISCUSSIONS

In this section we display the graphs of the results obtained as follows:

Figure 2 shows a parabolic velocity profile across the channel with maximum velocity along the centre line of the channel.

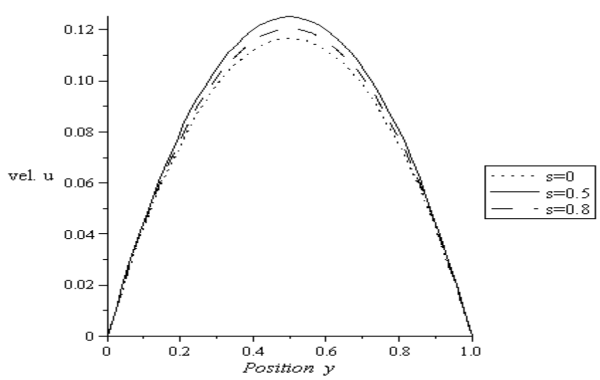

Figure 2: Graph of Velocity u(y) against Position y at Various Values of s.

The case of $s=0(s=1/\sqrt{Da})$ coincides with the well known plane Poiseulle flow. It was observed that fluid velocity decreases as porous media permeability decreases (s increases).

Figure 3 shows the temperature profiles across the channel for different axial distance with no effect of Br, (i.e Br=0).

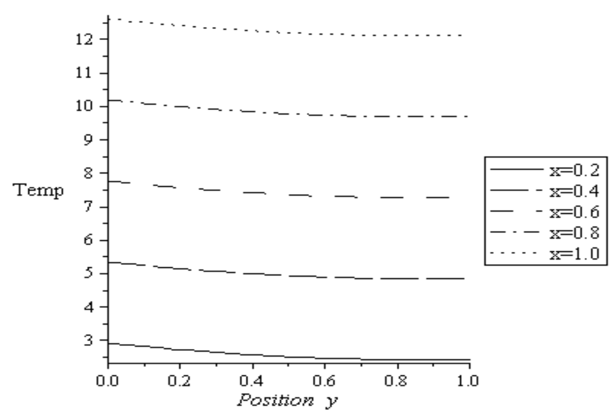

Figure 3: Graph of Temperature θ(y) vs. Position y at different values of x with s=0.3, Br=0.

We observed that the fluid temperature increases downstream (i.e., axially) and decreases transversely across the channel when the Brinkman number, Br is zero.

In Figure 4 we observed that the fluid temperature decreases in the transverse direction and increases with a decrease in porous medial permeability (s increases) at Br=0.

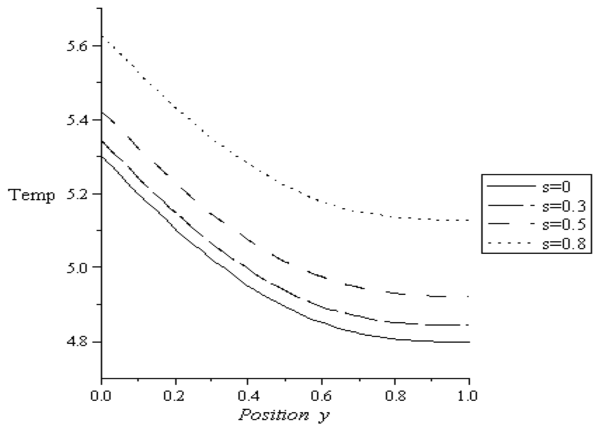

Figure 4: Graph of Temperature θ(y) against Position y at Different Values of s with x=0.4, Br=0.

Figure 5 showed the effect of Brinkman number, Br on the temperature profiles across the channel for different axial distance. It was shown that the higher values of Br decreases the temperature downstream (i.e., axially) and decreases transversely across the channel while in Figure 6, it was revealed that the fluid temperature decreases in the transverse direction and increases with a decrease in porous media permeability (s increases) as the Binkman number increases.

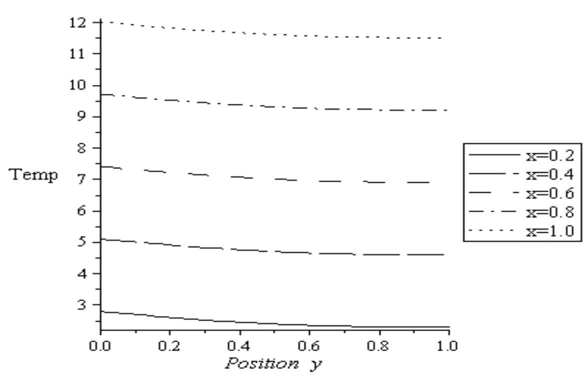

Figure 5: Graph of Temperature θ(y) against Position y at Different Values of x with s=0.3, Br=0.6

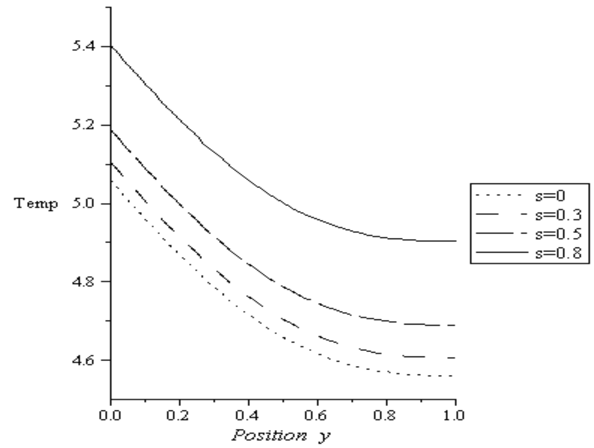

Figure 6: Graph of Temperature θ(y) against Position y at Different Values of s with x=0.4, Br=0.6

Generally, in Figure 7, as Br increases the temperature profile decreases both axially and transversely across the channel.

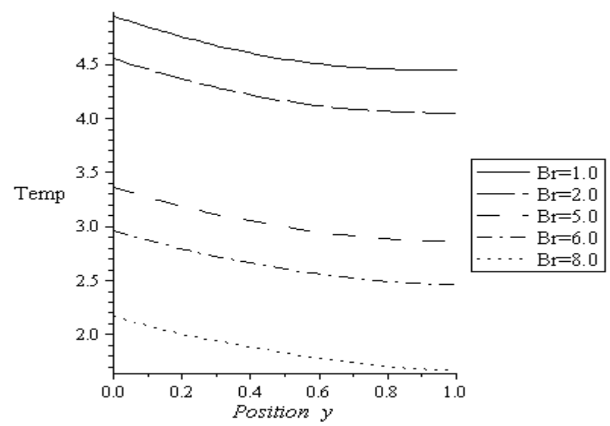

Figure 7: Graph of Temperature θ(y) against Position y at Different Values of Br with s=0.3, x=0.4.

CONCLUSION

This paper presents the effect of viscous dissipation on temperature profile of the flow in a channel filled with saturated porous media using method of separating the variables and data was generated using Pascal programming language and Maple software packages.

Velocity profile was obtained and used to compute the temperature profiles.

In general, Figure 7 clearly revealed that the fluid temperature decreases faster at higher values of Brinkman number, Br, both axially and transversely across the channel as compared to that of Makinde et al. [4].

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