

The Influence of Compressibility on Economic Pipe Diameter for Viscous Flow.

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ABSTRACT

Experimental inferences drawn by researchers on fluid flow have indicated the dependence of nature of flow as well as fluid flow parameters in ducts/pipes on the physical properties of fluid. For turbulent flow condition, authors have developed procedures for optimization of conduct size and its dependence on fluid compressibility have been reported. In this work, the iterative optimization procedure for laminar flow in pipeline is reviewed. The resulting optimum function was simulated and validated to generate database for the EXCEL[®] package, which was used to evolve quantitative relationship between fluid compressibility and optimum pipe diameter. Results obtained revealed a linear dependence of the form: $\gamma = m.d_{ecn} + c$, a situation similar to what was reported for turbulent flow where fluids with higher compressibility require larger optimum pipe size.

(Keywords: laminar flow, compressibility, economic pipe diameter, optimization, engineering)

INTRODUCTION

In recent times, for the economic approach to design of processes as well as engineering systems, machinery and structures have received the utmost attention by researchers. The apparent successes recorded through the engineering economy concept had been aided by available optimization techniques, which are based on minimum cost per unit of time or maximum profit per unit of production (*Peters and Timmerhaus, 1968*).

The former have been employed by authors (*Alamu, Adigun, and Durowoju, 2002; Ojediran, 2003*) for optimization of turbulent fluid flow in pipelines. The reported dependence of fluid flow behaviour on inherent physical properties of fluid prompted other researchers, notably *Akintola (2003) and Alamu, Adekunle, and Odewole, (2003)* to investigate the influence of fluid properties such as compressibility and density on the optimum diameter of pipes for turbulent flow.

In pipe flow problems, pipe sizes are selected based on the design criteria and economic considerations (*Akintola and Alamu, 2002*). The capital cost of a pipe run increases with diameter, whereas the pumping cost decrease with increasing diameter.

Selection of optimum pipe diameter for any type of flow; turbulent or laminar, has therefore been seen as a vital economic decision. To achieve this, optimization procedures were proposed by other authors (*Peters and Timmerhaus, 1968*), and were subsequently adopted by later researchers to determine economic pipe sizes for fluid flow using computer simulation (*Alamu et al., 2002; Ojediran, 2002; Akintola, 2003*).

The present work presents a review of theories of fluid dynamics and fluid flow cost concept to complement the earlier works of *Akintola (2003)* by extending the investigation of the effect of compressibility of fluid on economic pipe diameter to laminar flow cases, characterized with low Reynold's number ($N_{RE} < 2000$).

MATERIALS AND METHODS

It has been reported by authors (*Peters and Timmerhaus, 1968*) that for most types of pipe, a plot of the logarithm of the pipe diameter versus the logarithm of the purchase cost per unit length of pipe is essentially a straight line. Hence the piping cost, incorporating capital and maintenance charges, has been expressed as:

$$C_{\text{piping}} = Xd^n(1 + F)K_F \quad (1)$$

where:

X = cost per unit length of pipe, (Nmm^{-1})
 K_F = maintenance, and capital charges expressed as a fraction of initial cost for completely installed pipe
 D = pipe diameter, (mm)
 F = constant (fittings and installation cost / cost for new pipe)
 n = constant (pipe material dependent).

Another author has indicated that the constant n is a function of the current piping cost (*Sinnot, 1993*). For steel pipes, this constant has been approximated to 1.5 if $d \geq 25.0\text{mm}$ and 1.0 if $d < 25.0\text{mm}$ [5]. Also, for constant F , authors (*Peters and Timmerhaus, 1968; Alamu et al, 2002, Akintola, 2003*) have used the ratio 7:5

One of the most widely used equations for pipe flow, which satisfied experimental inferences, is the Darcy-Weisbach formula expressed as: (*Theodore and Lionel, 1967; Douglas et al, 1995*):

$$h_f = \frac{fLV^2}{2gd} \quad (2)$$

where,

f = fanning friction factor,
 L = length of pipeline, (m),
 V = linear fluid velocity, (ms^{-1}),
 d = diameter of pipe, (m)
 g = acceleration due to gravity, (ms^{-2}),
 h_f = frictional head loss, (m)

The friction factor is dependent on the nature of flow. Expressions therefore abound in literatures (*Peters and Timmerhaus, 1968; Kurmi, 1991; Sinnot, 1993*) relating the friction factor with

Reynolds number. For laminar or streamline flow, fanning friction factor has been expressed as:

$$f = \frac{16}{N_{RE}} \quad (3)$$

where, N_{RE} = Reynold's number

Equation (3) above is valid for flow conditions, characterized by ($N_{RE} < 2000$). The Reynold's number can be written in the form:

$$N_{RE} = \frac{Vd\rho}{\mu} \quad (4)$$

where,

μ = fluid viscosity, (Nsm^{-2}),
 ρ = fluid density, (kgm^{-3})

The linear velocity of the fluid can be expressed as:

$$V = \frac{4Q}{\pi d^2} \quad (5)$$

Experimental inferences of Osborne Reynolds have been reported (*Coulson and Richardson, 1996*) to postulate the dependence of flow pattern in ducts on the physical properties of fluid. Such properties include density, viscosity, and compressibility.

The compressibility, γ , of any type of fluid may be obtain from:

$$\gamma = \frac{1}{\rho} \quad (6)$$

Equations (3), (4), (5), and (6) can be combined to obtain the friction factor for streamline flow as:

$$f = \frac{4\pi\gamma\mu d}{Q} \quad (7)$$

Using Equation (7), the fanning pressure drop equation (*Sinnot, 1993; Akintola et al., 2003*) for laminar flow becomes:

$$P_d = 1.3486074 \times 10^{13} Q\mu d^{-4} \gamma^{1.84} \quad (8)$$

For flow through pipes of constant diameter, Sinnott (1993) expressed the annual pumping cost as:

$$C_{pm} = \frac{P_d Q t C_e}{\rho \eta} \quad (9)$$

where;

- P_d = pressure drop, (kNm^{-2})
- Q = fluid flow rate (kgs^{-1}),
- C_e = cost of electrical energy, (₦ kWhr^{-1}),
- t = operational hours per year; (hr. yr^{-1}),
- ρ = fluid density, (kgm^{-3})
- η = efficiency of motor and pump, (%)

Akintola (2003) and Alamu et al. (2003) have used Equation (9) above in the economic analysis of turbulent and viscous flow in pipes, respectively. Equations (6), (8), and (9) can be combined to obtain:

$$C_{pm} = 1.3486074 \times 10^{13} Q^2 \mu C_e t d^{-4} \eta^{-1} \gamma^{2.84} \quad (10)$$

The addition of Equations (1) and (10) gives the total annual cost of pumping and piping installation for the pipe flow system. Thus:

$$C_T = \frac{1.3486074 \times 10^{13} Q^2 \mu C_e t \gamma^{2.84}}{\eta d^4} + (1 + F) X d^n K_F \quad (11)$$

The pipe diameter d in Equation (11) for which the expression has minimum value represents the economic pipe diameter. In this work, the diameter is determined through computer simulation of the foregoing equations using FORTRAN 77 computer codes. With an initial guess value of the internal diameter of pipe, the nature of flow is determined from Equation (4) and the total annual cost evaluated using Equation (11). This procedure is repeated with increase in the value of the pipe diameter, and the total cost determined in each case until the total cost function passes through a minimum point.

The above problem has been solved by Durowoju and Alamu (2003) who adopted the analytical optimization approach to obtain an expression for

the economic pipe diameter (transformed to a function of compressibility) as:

$$d = 557.6851829 \left[\frac{Q^2 \mu C_e t \gamma^{2.84}}{(1 + F) X K_F \eta} \right]^{\frac{1}{5}} \quad (12)$$

SOFTWARE ASSESSMENT

Laminar flow case study due to Durowoju and Alamu (2003) (Case 1), applicable under ordinary industrial condition (Table 1) was used in testing and ascertaining the validity of the developed program. The above technique was adopted for different hypothetical values of fluid compressibility in the laminar flow case studies of Akintola (2003) (Case 2) and Durowoju and Alamu, (2003). In each case, the economic pipe diameter was determined. Subsequently, quantitative relationship between fluid compressibility and the economic pipe diameter was sought through the EXCEL package for the cases considered.

Case Studies

Case 1

- $\gamma = 1.111 \times 10^{-3} \text{m}^3 \text{kg}^{-1}$,
- $Q = 4.2 \text{kgs}^{-1}$,
- $K_F = 7.5\%$,
- $F = 1.4$,
- $C_e = \text{₦}30 \text{kWhr}^{-1}$,
- $t = 8000 \text{hryr}^{-1}$,
- $\eta = 0.6$,
- $\mu = 10 \text{mNsm}^{-2}$,
- $X = \text{₦}4.00 \text{mm}^{-1}$,
- $d_0 = 28.90 \text{mm}$,
- $\delta d = 0.05 \text{mm}$

Case 2

- $\gamma = (9.8039 - 11.3636) \times 10^{-4} \text{m}^3 \text{kg}^{-1}$,
- $Q = 4.0 \text{kgs}^{-1}$,
- $K_F = 7.5\%$
- $F = 1.4$,
- $C_e = \text{₦}30 \text{kWhr}^{-1}$,
- $t = 8024 \text{hryr}^{-1}$,
- $\eta = 0.5$,
- $\mu = 11.5 \text{mNsm}^{-2}$,
- $X = \text{₦}4.00 \text{mm}^{-1}$,
- $d_0 = 29.5 \text{mm}$,
- $\delta d = 0.05 \text{mm}$

RESULTS AND DISCUSSION

The output of the program for the flow data of Case 1 above is presented in Table 1. The results gave good agreement with the manual computations of Durowoju and Alamu (2003) as well as calculated values obtained through the analytical approach [Equation (12)]. The pipe diameter, corresponding to the least total cost, as evident in Table 1 and graphically illustrated in Figure 1, is 30.4mm. The same value was obtained by Durowoju and Alamu (2003) while Equation (12) gave 30.41mm.

Table 1 : Output of the Computer Program for Case 1

SN.	PIPE DIA (mm)	PRESSURE DROP (KN/sq.m)	PUMPING COST (=N=)	PIPING COST (=N=)	TOTAL COST (=N=)
1	29.30	2.8175	5259.26	11419.15	16678.42
2	29.35	2.7983	5223.52	11448.40	16671.91
3	29.40	2.7793	5188.07	11477.66	16665.74
4	29.45	2.7605	5152.93	11506.96	16659.88
5	29.50	2.7418	5118.08	11536.27	16654.35
6	29.55	2.7233	5083.53	11565.61	16649.14
7	29.60	2.7050	5049.27	11594.98	16644.25
8	29.65	2.6868	5015.29	11624.37	16639.67
9	29.70	2.6687	4981.61	11653.79	16635.40
10	29.75	2.6508	4948.20	11683.23	16631.43
11	29.80	2.6331	4915.08	11712.70	16627.77
12	29.85	2.6155	4882.23	11742.19	16624.41
13	29.90	2.5980	4849.65	11771.70	16621.35
14	29.95	2.5807	4817.35	11801.24	16618.59
15	30.00	2.5636	4785.31	11830.81	16616.12
16	30.05	2.5465	4753.54	11860.40	16613.94
17	30.10	2.5297	4722.04	11890.01	16612.05
18	30.15	2.5129	4690.79	11919.65	16610.44
19	30.20	2.4963	4659.80	11949.31	16609.11
20	30.25	2.4799	4629.07	11979.00	16608.07
21	30.30	2.4635	4598.59	12008.71	16607.30
22	30.35	2.4473	4568.36	12038.45	16606.81
23	30.40	2.4313	4538.38	12068.21	16606.59
24	30.45	2.4153	4508.65	12098.00	16606.64
25	30.50	2.3995	4479.15	12127.81	16606.96
26	30.55	2.3839	4449.90	12157.64	16607.54
27	30.60	2.3683	4420.89	12187.50	16608.39
28	30.65	2.3529	4392.11	12217.38	16609.50
29	30.70	2.3376	4363.57	12247.29	16610.86
30	30.75	2.3225	4335.26	12277.22	16612.48
31	30.80	2.3074	4307.17	12307.18	16614.35
32	30.85	2.2925	4279.32	12337.16	16616.48
33	30.90	2.2777	4251.69	12367.17	16618.85
34	30.95	2.2630	4224.28	12397.20	16621.48
35	31.00	2.2484	4197.09	12427.25	16624.34
36	31.05	2.2340	4170.12	12457.33	16627.45
37	31.10	2.2197	4143.37	12487.43	16630.80
38	31.15	2.2054	4116.83	12517.56	16634.39
39	31.20	2.1913	4090.50	12547.71	16638.21
40	31.25	2.1774	4064.39	12577.88	16642.27

Also, results obtained revealed a decrease in pressure drop with increase in pipe diameter as theoretically implied by Equation (8). Pumping cost also decreased while piping cost increases with increase in pipe diameter as earlier observed by Akintola (2003) and Alamu and Taiwo (2003). The above agreements offer some level of justification for the validity of the developed software for base line economic analysis of laminar flow problems in pipelines. However, this does not preclude the judgments of the engineer in respect of other practical considerations which may recommend values other than the theoretically predicted economic values.

Using the two sets of laminar flow cases presented, a total of 30 continuous flow problems were generated, representative of fluids with 15 different values of compressibility [(9.8039-11.3636) $\times 10^{-4} \text{m}^3/\text{kg}^{-1}$] under two distinct flow conditions. Data from these were fed as input into the validated computer program to generate output, similar to Table 1; but, summarized as shown in Table 2. A graphical illustration of the results is presented in Figure 2.

Table 2: Variation of Economic Pipe Diameter with Fluid Compressibility for the cases Considered.

S/N	FLUID COMPRESSIBILITY $\times 10^{-4} (\text{m}^3/\text{kg})$	ECONOMIC PIPE DIAMETER	
		(case 1)	(case 2)
		(mm)	
1	9.80392	28.50	29.7
2	9.90099	28.65	29.85
3	10.00000	28.80	30.00
4	10.10101	28.95	30.20
5	10.20408	29.10	30.35
6	10.30927	29.25	30.50
7	10.41666	29.40	30.65
8	10.52631	29.60	30.85
9	10.63829	29.75	30.00
10	10.75268	29.90	31.15
11	10.86956	29.05	31.35
12	10.98900	30.25	31.50
13	11.11111	30.40	31.70
14	11.23595	30.60	31.90
15	11.36363	30.75	32.05

As evident from Figure 2, the economic pipe diameter increases with fluid compressibility. This presents the same theoretical inference drawn by Akintola (2003) for fluid flow where Reynold's number are higher ($N_{RE} \geq 2000$).

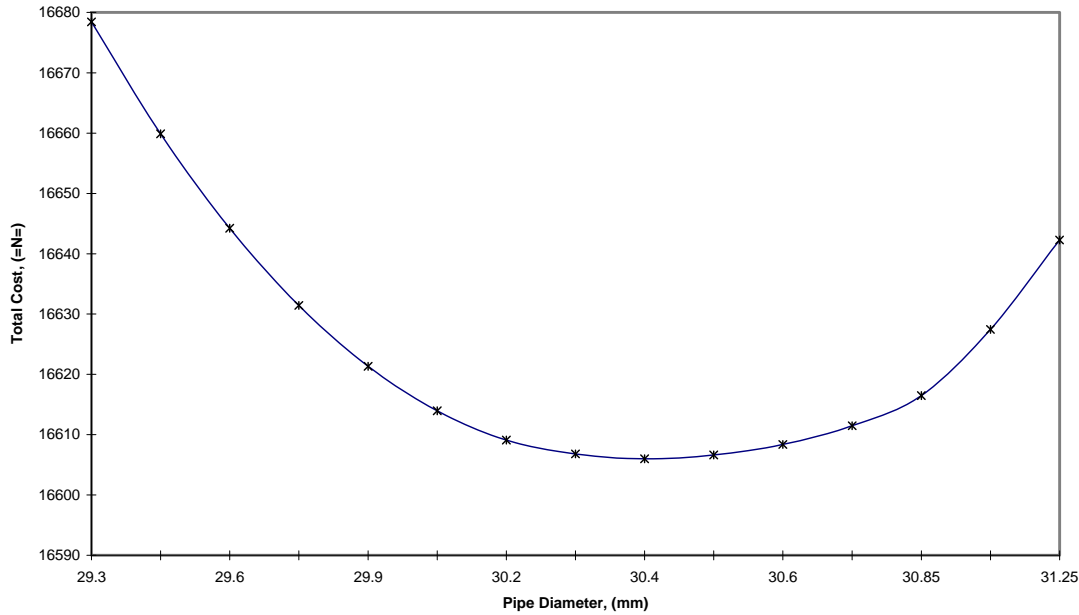


Figure 1: Variation of Annual Total with Pipe Diameter for Laminar Flow in Pipes.

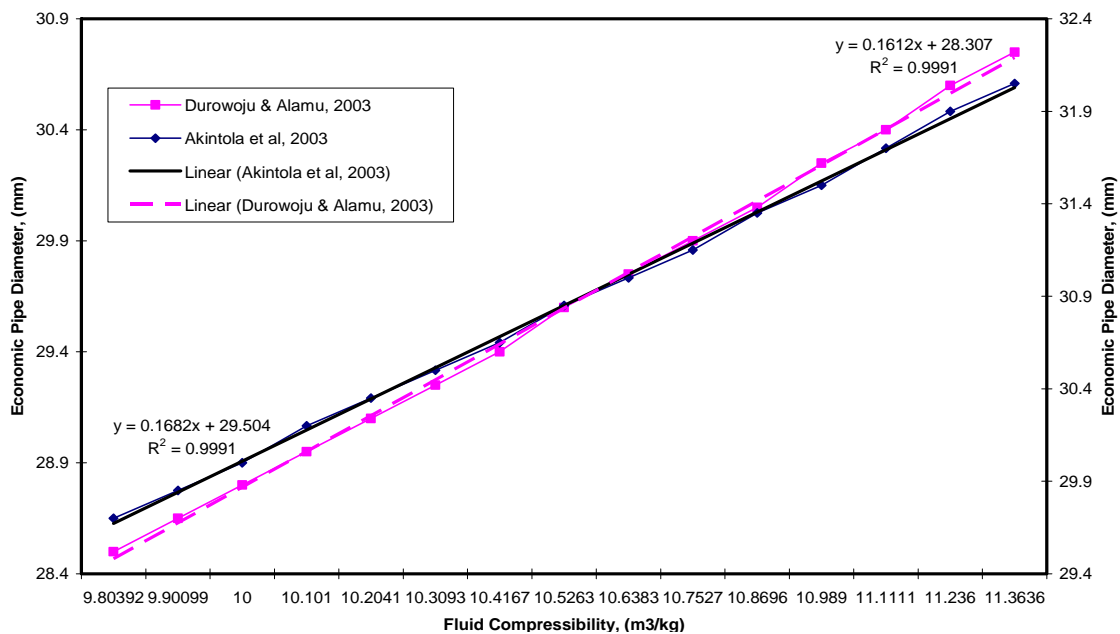


Figure 2: Economic Pipe Diameter Variation with Fluid Compressibility.

This suggests that the effect of fluid density on the economic pipe size is independent of the Reynold's number, and hence, nature of flow.

In quantitative terms, as revealed in Figure 2 through the EXCEL SOFTWARE, the relationship

is linear and of the form: $\gamma = m.d_{ecn} + c$, where, m and c are real values characteristic of the specific flow. For the cases considered in this work, the respective values are as shown in Figure 2.

CONCLUSIONS

Analysis of economic pipe diameter for laminar flow in pipelines was simulated using iterative technique. Results obtained from the validation of the developed software showed that economic pipe diameter increases with fluid compressibility. The optimum pipe diameter was also found to be independent of the Reynold's number.

From the development and evaluation of PKO biodiesel carried out, the following conclusions can be drawn:

- The transesterification process carried out using 100g PKO, 20.0g ethanol, 1.0% KOH (by weight of PKO) at 60°C reaction temperature and 90 minutes reaction time yielded 95.4g PKO biodiesel.

- At 15.56°C, specific gravity of PKO biodiesel is 1.033958 times that of fossil diesel.
- At 40°C, the PKO biodiesel had 85.06% reduction of viscosity over its raw vegetable oil.
- Higher pour (2°C), cloud (6°C) and flash (167°C) points were obtained for the PKO biodiesel compared to -12°C, -16°C and 74°C respectively obtained for D2.
- The limited fuel characterization carried out demonstrated that the PKO biodiesel produced can successfully fuel a diesel engine.

COMPUTER PROGRAM

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C PROGRAM: MODIFIED ECONOMIC PIPE DIAMETER FOR LAMINAR FLOW '
C
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IMPLICIT REAL*8(A-H,O-Z)
DIMENSION D(60),Pd(60),Cpu(60),Cpi(60),CTOTAL(60),Re(60)
OPEN(UNIT=8,FILE='FORUM.OUT')
WRITE(8,*)
C ***** INTERACTIVE DATA INPUT
PAI=22.0/7.0
WRITE(*,*)'Input the compressibility of the fluid, (kg/cubic metre)'
READ(*,*)comp
WRITE(*,*)'Input the fluid viscosity,(Ns/sq.metre)'
READ(*,*)FV
WRITE(*,*)'Supply the fluid flowrate,(kg/s)'
READ(*,*)Q
WRITE(*,*)'Input the annual fixed charges including maintenance,
1expressed as a fraction of initial cost for completely installed p
2ipe'
READ(*,*)fk
WRITE(*,*)'Enter the cost of electrical energy,(=N=kWh)'
READ(*,*)eC
WRITE(*,*)'Input the operation time of the system,(hr/yr)'
READ(*,*)t
WRITE(*,*)'Enter the pumping plant efficiency'
READ(*,*)EP
WRITE(*,*)'Supply the purchase cost per metre lenght of a 1mm diam
3eter pipe (=N=)'
READ(*,*)X
WRITE(*,*)'enter ratio of total cost for fittings and installation
4to purchase cost for new pipe'
READ(*,*)F
WRITE(*,*)'Input a guess value for the pipe diameter'
READ(*,*)D(1)
WRITE(*,*)'Supply a small increment in pipe diameter'
READ(*,*)XD
WRITE(8,*)'OUTPUT:ECONOMIC PIPE DIAMETER FOR LAMINAR FLOW'
WRITE(8,*)
WRITE(8,*)
5_
WRITE(8,7)
WRITE(8,9)
```

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WRITE(8,10)
WRITE(8,*)'_____
6_'
7 FORMAT(/,1X,'SN.',1X,' PIPE',3X,'PRESSURE',5X,'PUMPING',4X'PIPING'
8,5X,'TOTAL')
9 FORMAT(6X,'DIA',4X,'DROP',9X,'COST',7X,'COST',7X,'COST')
10 FORMAT(6X,'(mm)',3X,'(KN/sq.m)',4X,'(=N=)',6X,'(=N=)',6X,'(=N=)')

C ***** COMPUTATION
DO 30 I=1,40
D(I+1)=D(I)+XD
Re(I)=(4.0*Q)/PAI*FV*D(I)*comp
IF(Re(I) .GT. 2000.0) GO TO 50
Pd(I)=(D(I)**4.0)*(comp**(-1.84))
Pd(I)=1.3486074e13*Q*FV/Pd(I)
Cpu(I)=Pd(I)*Q*eC*t
Cpi(I)=Cpu(I)*comp/EP
IF(D(I) .GT. 25.0)THEN
Zn=1.5
ELSEIF (D(I) .EQ. 25.0)THEN
Zn=1.5
ELSE
Zn=1.0
ENDIF
Cpi(I)=X*(D(I)**Zn)
Cpi(I)=Cpi(I)*(1.0+F)*fK
CTOTAL(I)=Cpu(I)+Cpi(I)
C ***** Writing Result to File FORUM.OUT
WRITE(8,20)I,D(I),Pd(I),Cpu(I),Cpi(I),CTOTAL(I)
20 FORMAT(1X,I2,2X,F6.2,2X,F7.4,3X,F10.2,3X,F9.2,1X,F9.2)
30 CONTINUE
WRITE(8,*)'_____
8_'
WRITE(*,*)'Type EDIT FORUM.OUT for the result'
STOP
50 WRITE(8,*)'FLUID FLOW IS NOT VISCOUS'
STOP
END

```

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