

# Magnetics of an Idealized Asynchronous Reluctance Machine with no Moving Conductors.

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## ABSTRACT

The performance characteristics of an electrical machine is mainly dependent among other factors like the flux and mmf distributions in the machine; and hence its magnetics. An attempt is made in this paper to study the magnetics which underlie the performance characteristics of an idealized asynchronous reluctance machine with no moving conductors. The basis of the study is an idealized form of a two-stack machine, having continuously skewed windings with infinitesimally thin conductors located on the un-slotted stator side. The magnetics of the machine is studied for its various modes of operation, from standstill to full speed and the study show good agreement with the known characteristics behavior of the machine. It is shown that if the rotor has axial saliency instead of the usual circumferential saliency, that the machine will be propelled in the axial direction. It is further shown that the machine belongs to the class of machines known as the screw thread motor.

(Keywords: magnetomotive force, mmf, permeance, auxilliary transposed windings, skewed windings)

## INTRODUCTION

The cascade connection of induction motors had long made its debut as a means by which low speeds of operation can be obtained [1, 2]. Subsequently, there were rekindled interests in development of a single unit, self-cascaded machine from the point of view of reduced cost and improved performance [3]. The self-cascaded machine operates in a manner equivalent to the two machine arrangement. The windings in the separate stator and rotors are replaced by single stator-rotor unit. The work of Broadway [4] improved the industrial acceptance of cascade connection of induction motors for asynchronous operation. The scheme isolates the rotor low

frequency field component  $(1-2s)\omega_0$  from interfering with the primary supply system and the associated vibrations and noise were thus minimized. Other researchers, notably Agu [5], also configured the machine to operate asynchronously as coupled poly-phase reluctance machine. Subsequently, Agu et al. studied the generalized mechanism of torque production in coupled poly-phase reluctance machines without rotating windings using rotor permeance distribution function approach [6].

The Agu [5] configuration, unlike Broadway's [4] version, has no moving or rotating winding. The absence of rotating winding is an obvious advantage. In its basic form, it comprises two identical poly-phase reluctance machine with no moving conductors whose salient pole rotors are mechanically coupled together such that their d-axes are in space quadrature. The stator windings are integrally wound. However, each machine element has in fact two sets of identical stator windings known as the main and auxiliary windings. In the auxiliary winding, the coils are transposed between the two machine stacks. The stator and rotor are wound for the same pole number and both are star-connected.

A simple idealized configuration of the machine of n-stator and n-pole type may consist of a salient pole rotor (or its other equivalent forms) and two sets of winding on the stator, one set, the main winding skewed  $+180^\circ$  and the other set, the auxiliary winding skewed  $-180^\circ$ . An alternative configuration will result from say fixing the near end of the machine and then twisting the other end through  $-180^\circ$  such that the main winding is no longer skewed, while the rotor pole becomes skewed  $-180^\circ$  and the auxiliary winding becomes skewed  $-360^\circ$ . This continuously skewed machine in an idealization of a machine made of n identical stacks, with the axis of the stator windings shifting

$+\frac{\pi}{n}or-\frac{\pi}{n}$  radians as the case may be from

one stack to the next. If L is the total effective length of the machine, then the length of each

stack =  $\frac{L}{n}$  meters. As n tends to a large number,

the machine tends to the idealized machine. The main subject of this paper is to study the magnetics underlying the performance characteristics of an idealized asynchronous machine using the two-stack reluctance machine with no moving conductors as the basis of the study.

### PHYSICAL CONFIGURATION OF THE TWO-STACK MACHINE

In the two-stack machine, the rotor is made up of two identical equal halves, whose poles axes are  $\pi/2$  electrical radians out of phase. There are two physically isolated, but magnetically coupled, identical windings on the stator known as the main and auxiliary windings and sharing a common magnetic structure. The axes of the main windings are the same in both halves of the machine whereas the axes of the auxiliary windings are transposed (anti-series connection) in passing from one half of the machine to the other as shown in Figure 1, such that, if the mutual inductance between the main and auxiliary windings in one half of the machine is positive, in the other half, it will be negative. The mutual inductances are functions of the rotor angular position.

### ANALYSIS OF THE TWO-STACK MACHINE

The main winding is connected to the utility supply of  $\omega_0$  frequency, from which it drives its magnetizing current  $I_0$ . The magnetizing current produces magnetizing mmf which is usually assumed to be sinusoidally distributed in each stack of the machine and may be expressed as :

$$m_0 = M_0 \cos (\theta - \omega_0 t) \quad (1)$$

If at the instant  $t = 0$ , the d-axis of stack A machine rotor coincides with  $\theta=0$  location, the permeance presented to the mmf  $m_0$  by machine A stack assuming a two rotor will be given by:

$$P_A = P_0 + P_1 \cos 2(\theta - \omega t) \quad (2)$$

Where  $\omega$  is the speed of the rotor.

The permeance distribution presented by the machine B stack whose poles axis is  $\pi/2$  electrical radians displaced from stack A will be given by:

$$\begin{aligned} P_B &= P_0 + P_1 \cos 2(\theta - \omega t - \pi/2) \\ &= P_0 - P_1 \cos 2(\theta - \omega t) \end{aligned} \quad (3)$$

The flux density distribution in stack A, due to excitation mmf  $m_0$  may be expressed as :

$$\begin{aligned} B_A &= m_0 P \\ &= M_0 P_0 \cos (\theta - \omega_0 t) + \frac{1}{2} M_0 P_1 \cos (\theta + (\omega_0 - 2\omega)t) + \text{third space harmonics} \\ &= B_0 \cos (\theta - \omega_0 t) + B_1 \cos (\theta + (\omega_0 - 2\omega)t) + \text{third space harmonics} \end{aligned} \quad (4)$$

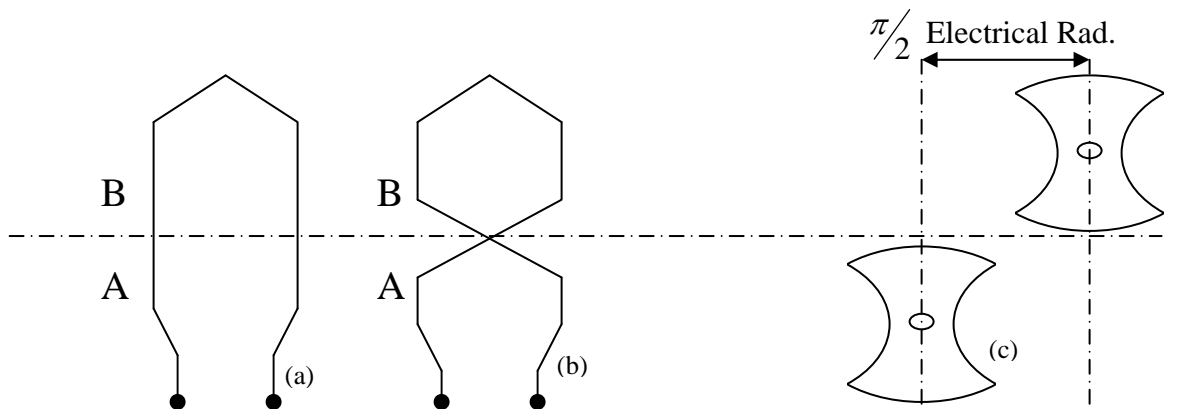


Figure 1: (a) Main Windings (b) Transposed Auxiliary Windings (c) Rotor Pole Axes in Space Quadrature.

Similarly, the flux density distribution in stack B is given by:

$$\begin{aligned}
 B_B &= m_0 P_B \\
 &= M_0 \cos(\theta - \omega_0 t) - \frac{1}{2} M_0 P_1 \cos(\theta + (\omega_0 - 2\omega)t) + \text{third space harmonics} \\
 &= B_0 \cos(\theta - \omega_0 t) - B_1 \cos(\theta + (\omega_0 - 2\omega)t) + \text{third space harmonics} \quad (5)
 \end{aligned}$$

The third space harmonics are eliminated if the stack machines are star connected.

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In stack A machine half, the two component fluxes of Equation 4 will induce their corresponding voltages in the main and auxiliary windings respectively. The induced voltages due to the first components is a positive sequence voltage at the supply frequency  $\omega_0$ . The induced voltage due to the second component is a negative sequence voltage at the frequency of  $(\omega_0 - 2\omega)$ .

Similarly, in stack B machine half, the two components of Equation 5 will induce their corresponding voltages in the main and auxiliary windings respectively. The induced voltage due to the first components is a positive sequence voltage at the supply frequency  $\omega_0$ , while the second will induce a negative sequence voltage in the main and auxiliary windings respectively at  $(\omega_0 - 2\omega)$  frequency. The voltages induced by the second components of Equation 4 and Equation 5

are in anti-phase due to the nature of the respective fluxes producing them ( $\pm B_1 \cos(\theta + (\omega_0 - 2\omega)t)$ ).

If  $e_0$  is the induced voltage due to the first components of Equations 4 and 5 and  $\pm e_2$  due to the second components, it follows that when the main windings of stack A and B machines are connected in series as shown in Figure 2, that the negative sequence voltages  $\pm e_2$  will cancel out while the positive sequence voltage will add up to  $2e_0$ . In the auxiliary winding however, the component voltages  $e_0$  will cancel out while the negative sequence voltages  $\pm 2e_2$  will add up in the transposed auxiliary winding to  $2e_2$ . Corollary, the flux inducing voltage in the main winding is obtained by the summation of Equations 4 and 5 and the flux inducing voltage in the auxiliary winding is obtained by the subtraction of Equation 5 from Equation 4 (Equation 4 - Equation 5).

The induced voltage in the auxiliary winding will circulate current in the auxiliary winding when it is closed and produce mmf whose distribution may be expressed as:

$$m_2 = M_2 \cos(\theta + (\omega_0 - 2\omega)t - \pi/2 - \phi_2) \quad (6)$$

Where  $\phi_2$  is the impedance angle of the auxiliary winding circuit. And if the circuit is purely inductive i.e.  $\phi_2 = \pi/2$ , the mmf will directly oppose the flux producing it, which is the second component of Equation 4 in accordance with Lenz's law.

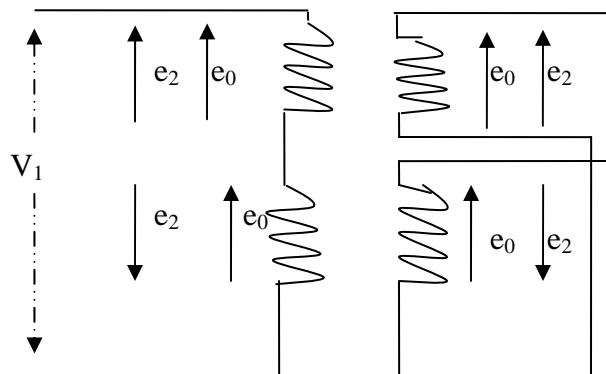


Figure 2: The Two-Stack Machine Showing Induced Component Voltages.

If Equation 6 stands for the mmf distribution in stack A half, its distribution in stack B half due to the transposition of auxiliary winding will be expressed as:

$$\begin{aligned} m_2 &= -M_2 \cos(\theta + (\omega_0 - 2\omega)t - \pi/2 - \phi_2) \\ &= M_2 \cos(\theta + (\omega_0 - 2\omega)t + \pi/2 - \phi_2) \end{aligned} \quad (7)$$

The flow of current in the auxiliary winding will cause the primary winding to draw an additional  $I_1$  from the utility supply to produce a counteracting mmf  $m_1$  which balances  $m_2$  and whose distribution may be expressed as:

$$m_1 = M_1 \cos(\theta - \omega_0 t + \pi/2 - \phi_1) \quad (8)$$

$\phi_1$  is the primary input power factor angle.

It will be shown later that the magnitudes of  $m_1$  and  $m_2$  are equal. However,  $m_1$  and  $m_2$  rotate in the opposite directions and hence may be expressed as:

$$|m_1| = -|m_2| \quad (9)$$

Consequently, Equation 8 can be re-written as:

$$\begin{aligned} m_1 &= -M_2 \cos(\theta - \omega_0 t + \pi/2 - \phi_1) \\ &= M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1) \end{aligned} \quad (10)$$

## TORQUE

Torque is in general proportional to the square of mmf. The torque of the machine is due to the interaction of rotating mmfs of the auxiliary and main winding currents in the air-gap. The interaction of the mmfs in stack A half say, may be expressed as:

$$\text{mmf}_A = M_2 \cos(\theta + (\omega_0 - 2\omega)t - \pi/2 - \phi_2) + M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1) \quad (11a)$$

$$= 2M_2 \sin\theta - \omega t - \left(\frac{\theta_2 + \theta_1}{2}\right) \cos\left(\omega_0 t - \omega t - \left(\frac{\theta_2 - \theta_1}{2}\right)\right) \quad (11b)$$

Equation 11b is a complex wave and the product of two waves. One component is a two-pole field as the rotor and moves in step with the rotor, with its axis leading that of the rotor pole by  $\delta =$

$$\frac{\theta_2 + \theta_1}{2}.$$

This component is responsible for torque production in stack A half as it propels the rotor in the forward positive direction at speed  $\omega$ . The amplitude of the torque producing component is modulated by the second component which is at the slip speed of  $(\omega_0 - \omega)$ . It can be readily shown that the resultant mmf in the air gap of stack B half may be expressed as:

$$\text{mmf}_B = 2M_2 \cos\theta - \omega t - \left(\frac{\theta_2 - \theta_1}{2}\right) \sin\left(\omega_0 t - \omega t - \left(\frac{\theta_2 - \theta_1}{2}\right)\right) \quad (12)$$

The first component of Equation 12 is the torque producing component whose amplitude is modulated by the second component. The two torque producing mmfs in stack A and B halves of the machine act in time quadrature; such that when the mmf and hence torque is maximum in stack A half, it is minimum in stack B, while the net average torque remains constant. Furthermore, the torque of the machine is synchronous in nature even though the machine operates asynchronously; since the rotating airgap mmf imparting torque on the rotor moves in step with the rotor while operating asynchronously.

## PRIMARY CURRENT AND MMF AT STANDSTILL ( $\omega = 0$ )

It is only the magnetizing mmf  $m_0$  that exists in the machine until the auxiliary winding circuit is closed, which causes the circulation of current in the auxiliary winding. The current flow in the auxiliary winding causes main winding to draw an additional current from the supply for ampere turns balance with the auxiliary winding. Let the mmf of the additional primary current be  $m_1$ . The auxiliary winding mmf  $m_2$  in stack A may be expressed as:

$$B_A = m_2 (P_0 + P_1 \cos 2\theta) \quad (13)$$

For stack B, it is expressed as:

$$B_B = -m_2 (P_0 - P_1 \cos 2\theta) \quad (14)$$

Thus, the overall flux density of the auxiliary winding linking the primary winding is the sum of Equation 13 and 14 and given by:

$$B_{TA} = 2m_2 P_1 \cos 2\theta \quad (15)$$

The flux density distribution due to the additional primary mmf  $m_1$  is given by:

$$B_{AP} = m_1 (P_0 + P_1 \cos 2\theta) \text{ in Stack A} \quad (16)$$

$$\text{and } B_{BP} = m_1 (P_0 - P_1 \cos 2\theta) \text{ in stack B} \quad (17)$$

Thus, the overall flux density of the primary winding linking the auxiliary winding will be the difference of Equations 16 and 17 and given by:

$$B_{TP} = 2m_1 P_1 \cos 2\theta \quad (18)$$

At equilibrium condition, the overall flux of the auxiliary winding linking the primary winding balances the overall flux of the primary winding linking the auxiliary winding and hence,

$$B_{TA} = B_{TP} \quad (19a)$$

$$\therefore |m_1| = |m_2| \quad (19b)$$

The two mmfs  $m_1$  and  $m_2$  also rotate at the same speed although in opposite directions [6]. The torque and location of the point of action of the torque mmf on the rotor is essentially similar to what obtains while running. The important difference occurs in the power factor angle of the auxiliary winding circuit, in which the frequency is now  $\omega_0$  instead of  $(\omega_0 - 2\omega)$ .

### SYNCHRONOUS OPERATION (at $\omega = \frac{\omega_0}{2}$ )

The frequency of the induced current in the auxiliary winding is  $(\omega_0 - 2\omega)$ . When the rotor runs at  $\omega = \omega_0/2$ , the frequency will be zero. Since the magnitude of the induced emf is proportional to the frequency, it follows that at  $\omega = \omega_0/2$ , there will be no induced voltage and current in the auxiliary winding and hence, zero torque.

If direct current is supplied externally to the auxiliary winding at this speed, the machine will operate at the synchronous speed [5]. With direct current injection into the auxiliary winding, its mmf will be stationary in nature. However, as the rotor rotates at  $\omega = \omega_0/2$  and the applied field from the supply rotates at  $\omega_0$ , it will be observed that although the DC excitation field is stationary; it moves with the speed  $-\omega_0/2$  relative to the rotor and the primary field moves with the speed  $+\omega_0/2$  relative to the rotor. There will therefore be a torque whose direction depends upon the

direction of the excitation mmf. The primary (main winding) fluxes due to the magnetizing mmf  $m_0$  at  $\omega = \omega_0/2$  are given respectively for stack A and B as:

$$m_0(P_0 + P_1 \cos 2(\theta - \frac{\omega_0 t}{2})) = M_0 P_0 \cos(\theta \omega_0 t) + (\frac{m_0 P_1}{2}) \cos \theta + \text{third space harmonics} \quad (20)$$

$$m_0(P_0 + P_1 \cos 2(\theta - \frac{\omega_0 t}{2})) = M_0 P_0 \cos(\theta - \omega_0 t) - (\frac{m_0 P_1}{2}) \cos \theta + \text{third space harmonics} \quad (21)$$

The primary excitation fluxes linking the auxiliary winding is given by the difference of Equations 20 and 21 (ie  $M_0 P_1 \cos 2\theta$ ), which is a stationary flux and therefore will induce no emf in the auxiliary winding. Therefore, there is no auxiliary winding induced currents and hence mmf  $m_2$  is zero and so is the electromagnetic torque at  $\omega = \omega_0/2$ . By supplying the auxiliary winding with external direct current, mmf  $m_2$  is thus provided. The DC field mmf due to  $m_2$  may be expressed as:

$$m_2 = \pm M_2 \cos \theta \quad (22)$$

The positive sign is for the distribution in stack A half, while the negative sign is for the distribution in stack B.

The flux density distribution in stack A half is given by:

$$B_A = m_0(P_0 + P_1 \cos 2(\theta - \frac{\omega_0 t}{2})) \quad (23)$$

$$= M_2 P_0 \cos \theta + \frac{m_2 P_1}{2} \cos(\theta - \omega_0 t) + \text{third space harmonics} \quad (24)$$

If Equation 24 stands for mmf distribution in stack A half, for stack B, it will be given by

$$B_B = M_2 P_0 \cos \theta + \frac{m_2 P_1}{2} \cos(\theta - \omega_0 t) + \text{third space harmonics} \quad (25)$$

The flux density distribution due to the first components of Equations 24 and 25 will induce no emfs, being stationary in nature. The second components of Equations 24 and 25 which are in anti-phase will induce emfs in the transposed auxiliary windings which are additive. The induced emf may be expressed as:

$$e_{2a} = 2E_2 \cos(\omega_0 t - \pi/2) \quad (26)$$

The induced voltage will produce a circulating current in the auxiliary winding which in turn will produce a rotating mmf  $m_2$  whose distribution in stack A half say may be expressed as:

$$m_2 = M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_2) \quad (27)$$

and, for stack B,

$$m_2 = -M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1) \quad (28)$$

The current flow in the auxiliary winding will cause the primary winding to draw an additional current  $I_1$ , whose mmf distribution in both stacks may be expressed as:

$$m_1 = M_1 \cos(\theta - \omega_0 t + \pi/2 - \phi_1) \quad (28a)$$

$$= M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1) \quad (28b)$$

The resultant mmf in stack A half may then be expressed as;

$$\text{mmf}_A = M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1) + M_2 \cos(\theta - \omega_0 t - \pi/2 - \phi_1)$$

$$= 2M_2 \sin(\theta - \omega_0 t - (\frac{\phi_1 - \phi_2}{2})) \cos(\frac{\phi_2 - \phi_1}{2}) \quad (29)$$

The first component of equation 29 is a two-pole field which rotates at the same speed as the primary applied field  $\omega_0$  and leads the rotor pole axis by  $\delta = (\frac{\phi_2 - \phi_1}{2})$  and will pull the rotor into synchronism. However the amplitude of the torque producing component will be modulated by the second component  $\cos(\frac{\phi_2 - \phi_1}{2})$ .

Alternatively, if the primary winding is fed through a stationary commutator and rotating brush gear and slip rings, and the auxiliary winding fed with DC, an operation similar to that of a conventional DC machine will result [5]. If salient poles are, however, introduced in the bore of the stator stacks, with axes corresponding with that of the DC field of the auxiliary winding, the machine will run as a salient pole synchronous machine or as a synchronous reluctance machine if the DC is removed [7].

## SUPER SYNCHRONOUS OPERATION ( $\omega > \omega/2$ )

The machine is a half speed machine with its synchronous speed  $\omega = \frac{\omega_0}{2}$ . The per unit slip  $s^1$  is given by:

$$s^1 = \frac{\frac{\omega_0}{2} - \omega}{\frac{\omega_0}{2}} \quad (30)$$

$$= (2s - 1) \quad (31)$$

$$\text{Where } s = \frac{\omega_0 - \omega}{\omega_0} \quad (32)$$

The slip is negative for  $s$  less than  $1/2$  and greater than  $-1$  ( $-1 < s < 1/2$ ). Within this range, the magnetizing flux density inducing emf in the auxiliary winding will be rotating in the positive anti-clockwise sense. The auxiliary winding mmf also rotates in the positive anti-clockwise sense. The electromagnetic torque developed will oppose the direction of motion and the machine will require external mechanical drive (generator operation).

## SYNCHRONOUS OPERATION WITH DUAL EXCITATION ( $\omega = \omega_0$ )

Suppose the rotor is brought up to synchronous speed of the applied field  $\omega_0$  by an auxiliary machine with the main and auxiliary windings connected to the supply. The two windings will draw equal currents if they have the same number of turns and produce mmfs  $m_1$  and  $m_2$ . The speed of both fields are equal in magnitude and opposite in direction. The mmfs will support each other in stack A half of the machine say and neutralize each other in stack B half, because of the transposition of the auxiliary winding. Torque will therefore be developed in stack A half and stack B half having zero net flux will be idling. In this mode, the motor will operate as a synchronous reluctance machine utilizing one stack of the coupled machines. Torque may, however, be distributed over both stack depending on the coincidence of the primary and auxiliary windings phase connections in parallel. Inequality in the number of turns between the main and auxiliary windings will lead to synchronous instability.

## THE MULTI – STACK MACHINE

Consider a multi-stack machine comprising  $n$  identical stacks with an overall length of  $L$  units. The shift in the auxiliary stator winding axis between stacks in a clockwise manner is  $\frac{2\pi}{n}$  electrical radians. The shift in the rotor pole axes between stacks in the same clockwise manner is  $\frac{\pi}{n}$  electrical radians as shown in Figure 3. A two-pole rotor is considered.

The air-gap permeance distribution of the first stack of the  $n$  stack machine may be expressed as:

$$P_1' = P_0 + P_1 \cos 2(\theta - \omega t) \quad (33a)$$

For the  $K^{\text{th}}$  stack, it will be given by,

$$P_k = P_0 + P_1 \cos 2\left(\theta - \omega t - \frac{k-1}{n} \pi\right) \quad (33b)$$

The usual sinusoidally distributed magnetizing mmf is assumed to be the same for all stacks and given by:

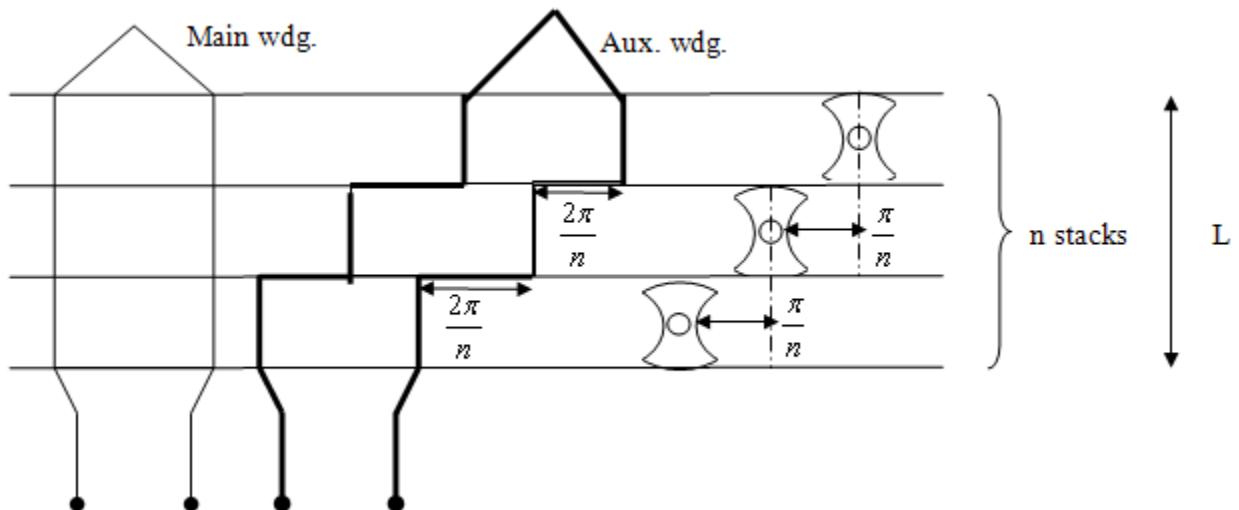
$$m_0 = M_0 \cos(\theta - \omega_0 t) \quad (34)$$

The flux density distribution in the  $K^{\text{th}}$  stack is given by:

$$B_k = m_0 P_k = M_0 P_0 \cos(\theta - \omega_0 t) + \frac{M_0 P_1}{2} \cos\left(\theta + (\omega_0 - 2\omega)t - \frac{k-1}{n} 2\pi\right) + \text{third space harmonics} \quad (35)$$

The second component of Equation 35 is a wave whose axis shifts  $\frac{2\pi}{n}$  rads between one stack and the next in the same manner as the axes of the phase of the auxiliary windings does. This component will link the auxiliary windings but not the primary windings. Consequently, it will induce voltage in the auxiliary winding. The induced voltage in the auxiliary winding will circulate current in the auxiliary winding whose mmf distribution may be expressed as:

$$m_2 = M_2 \cos\left(\theta + (\omega_0 - 2\omega)t - \frac{k-1}{n} 2\pi - \frac{\pi}{2} - \phi_2\right) \quad (36)$$



**Figure 3:** A Multi-Stack Machine Showing the Disposition of the Main and Auxiliary Winding and Rotor Poles.

The current flow in the auxiliary winding will cause the primary winding to draw an additional current from the source to produce an mmf which

balances that of the auxiliary winding and whose distribution may be expressed as:

$$m_1 = M_1 \cos(\theta - \omega_0 t + \frac{\pi}{2} - \phi_2) \quad (37a)$$

$$m_2 = M_2 \cos(\theta - \omega_0 t - \frac{\pi}{2} - \phi_1) \quad (37b)$$

The gross mmf in the air gap neglecting the magnetizing mmf may be expressed as:

$$mmf_k = M_2 \cos(\theta + (\omega_0 - 2\omega)t - \frac{k-1}{n} 2\pi - \frac{\pi}{2} - \phi_2) +$$

$$M_2 \cos(\theta - \omega_0 t - \frac{\pi}{2} - \phi_1) \quad (38)$$

$$= 2M_2 \sin(\theta - \omega t - \frac{k-1}{n} \pi - (\frac{\phi_2 - \phi_1}{2})) \cos((\omega_0 - \omega)t$$

$$- \frac{k-1}{n} \pi - (\frac{\phi_2 + \phi_1}{2})) \quad (39)$$

Equation 39 is a complex wave and the product of two waves. The first component is a wave which rotates in step with the rotor and spirals in the same manner as the rotor in the same forward direction and will consequently impart torque to the rotor. The focal point of torque will move from one stack to the other. The amplitude of the first component is modulated by the second component which pulsates at the slip frequency of  $(\omega_0 - \omega)$ .

## THE IDEALIZED MULTI – STACK MACHINE

When the number of stacks as described in section 9.0 becomes very large ( $n \rightarrow \infty$ ) while maintaining the same overall active length of machine at  $l$ , the length of each stack becomes

infinitesimally very small and  $\frac{k-1}{n} \rightarrow \frac{z}{l}$ , where

$z$  is the axial distance of the  $k^{\text{th}}$  stack from the start end of the machine and the machine tends to an idealized one. The air-gap permeance distribution of the idealized machine for continuously skewed rotor may be expressed as:

$$P_z = P_{oz} + P_{1z} \cos 2(\theta - \omega t - \frac{\pi z}{l}) \quad (40)$$

The flux density distribution in the air-gap may be expressed as:

$$B_z = m_0 P_z = M_0 \cos(\theta - \omega_0 t) \times (P_{oz} + P_{1z} \cos 2(\theta - \omega t - \frac{\pi z}{l})) \quad (41)$$

$$= M_0 P_{oz} \cos(\theta - \omega_0 t) + \frac{M_0 P_{1z}}{2} \cos(\theta + (\omega_0 - 2\omega)t - \frac{2\pi z}{l}) + \text{third space harmonics} \quad (42)$$

As discussed in the preceding sections, the second component of Equation 42 will link the auxiliary winding, having the same skew and hence will induce voltage in the auxiliary winding. The voltage induced in the auxiliary winding will produce a circulating current whose mmf distribution may be expressed as:

$$M_{2z} = M_2 \cos(\theta + (\omega_0 - 2\omega)t - \frac{2\pi z}{l} - \frac{\pi}{2} - \phi_2) \quad (43)$$

The load current in the auxiliary winding will cause additional current flow in the primary. The current in turn will produce an mmf  $m_1$  which balances the auxiliary winding mmf and whose distribution may be expressed as:

$$M_{1z} = M_1 \cos(\theta - \omega_0 t + \frac{\pi}{2} - \phi_1) \\ = M_2 \cos(\theta - \omega_0 t - \frac{\pi}{2} - \phi_1) \quad (44)$$

The interaction of the two mmfs  $m_{2z}$  and  $m_{1z}$  may be expressed as:

$$mmf_z = M_2 \cos(\theta + (\omega_0 - 2\omega)t - \frac{2\pi z}{l} - \frac{\pi}{2} - \phi_2) + \\ M_2 \cos(\theta - \omega_0 t - \frac{\pi}{2} - \phi_1) \\ = 2M_2 \sin(\theta - \omega t - \frac{\pi z}{l} - (\frac{\phi_2 + \phi_1}{2})) \cos((\omega_0 - \omega)t - \frac{\pi z}{l} - (\frac{\phi_2 - \phi_1}{2})) \quad (45)$$

As discussed in the preceding sections, Equation 45 comprises two components, one component which rotates in synchronism with the rotor and spirals in the same manner as the rotor and consequently will impart reluctance torque on the rotor. However, its amplitude is modulated by the

second component which pulsates at the frequency  $(\omega_0 - \omega)$ .

### IDEALIZED MACHINE WITH ARMATURE HAVING AXIAL SALIENCY

If the rotor is replaced with an armature which has an axial saliency as shown in Figure 4, the air-gap permanence distribution which results when the armature moves through the stator core may be expressed as:

$$P_{z1} = P_{z0} + P_{z1} \cos \frac{2\pi}{l} (z - vt) \quad (46)$$

Where  $v$  is the linear axial velocity of the armature and  $z$  is the distance along the axial length.

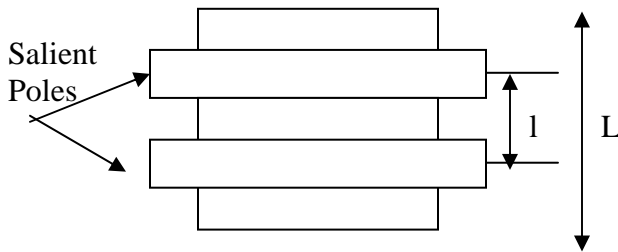


Figure 4: An Armature Having Axial Saliency.

The airgap flux density distribution due to the excitation mmf  $m_0$  may be expressed as:

$$\begin{aligned} B_z &= M_0 \cos(\theta - \omega t) \left[ P_{z0} + P_{z1} \cos \frac{2\pi}{l} (z - vt) \right] \quad (47) \\ &= M_0 P_{z0} \cos(\theta - \omega t) + \frac{M_0 P_{z1}}{2} \cos \left( \theta - \left( \omega_0 + \frac{2\pi}{l} v \right) t + \frac{2\pi}{l} z \right) + \\ &\quad \frac{M_0 P_{z1}}{2} \cos \left( \theta - \left( \omega_0 - \frac{2\pi}{l} v \right) t - \frac{2\pi}{l} z \right) \quad (48) \end{aligned}$$

The second and third components of Equation 48 are similar except that they are skewed in opposite directions (clockwise and anti-clockwise). An auxiliary winding skewed in the same sense as the second or third component of Equation 48 will capture the flux. Only one of the two possible skews is feasible from practical point of view. Suppose the skew is in the sense as the

second component, the second component of Equation 48 will link the auxiliary winding just as the equivalent for the rotary machine and induce a current of frequency  $(\omega_0 + \frac{2\pi v}{l})$  in the auxiliary winding and produce an mmf whose distribution which may be expressed as:

$$m_{z2} = M_2 \cos \left( \theta - \left( \omega_0 + \frac{2\pi}{l} v \right) t + \frac{2\pi}{l} z - \frac{\pi}{2} - \phi_2 \right) \quad (49)$$

The current flow in the auxiliary winding will cause the main (primary) winding to draw an additional current to produce an mmf which balances the auxiliary winding mmf and whose distribution may be expressed as:

$$\begin{aligned} m_{1z} &= M_1 \cos \left( \theta - \omega_0 t + \frac{\pi}{2} - \phi_1 \right) \\ &= M_2 \cos \left( \theta - \omega_0 t - \frac{\pi}{2} - \phi_1 \right) \quad (50) \end{aligned}$$

The resultant mmf in the air gap, neglecting the magnetizing mmf  $m_0$  may be expressed as:

$$\begin{aligned} \text{mmf}_{zR} &= M_2 \cos \left( \theta - \left( \omega_0 + \frac{2\pi}{l} v \right) t + \frac{2\pi}{l} z - \frac{\pi}{2} - \phi_2 \right) + \\ &\quad M_2 \cos \left( \theta - \omega_0 t - \frac{\pi}{2} - \phi_1 \right) \\ &= 2M_2 \sin \theta - \left( \omega_0 + \frac{\pi}{l} v \right) t + \frac{\pi}{l} z - \frac{(\phi_2 + \phi_1)}{2} \quad * \\ &\quad \cos \frac{\pi}{l} (z - vt - \Psi) \quad (51) \end{aligned}$$

Where  $\Psi = \frac{(\phi_2 - \phi_1)}{2} \cdot \frac{l}{\pi}$  and the synchronous linear velocity  $v = \frac{\omega_0 l}{2\pi}$

Equation 51 is a wave which spirals in the same manner as the rotor and moves in synchronism with the rotor, thereby producing a synchronous torque at asynchronous speed. The wave glides longitudinally along the surface of the machine while maintaining circular motion. The centre of gravity of the torque arising from this distribution glides over the rotor cyclically from one end to the other. When the auxiliary is removed and salient poles are introduced on the stator with axis coinciding with that of the auxiliary winding, the machine will be capable of running as a

synchronous reluctance machine like the screw thread machine. This machine belongs therefore to the same class of the screw-thread machine. From the analogy between the axial motion and the screw-thread motor, it is clear that if we have stator saliency in the rotating machine we can get synchronous reluctance operation at  $\omega = \frac{\omega_0}{2}$  with only the primary winding excited.

### COMPARISON OF THE SCREW-THREAD MOTOR AND THE IDEALIZED MACHINE WITH AXIAL SALIENCY

If along the axis parallel to the auxiliary windings, a pair of salient poles is constructed on the stator in the place where the DC would be, the resulting configuration is that of a screw thread motor [8].

The screw thread motor is a purely synchronous machine, requiring no auxiliary winding. However, the screw thread motor cannot operate asynchronously. It therefore follows that a combination of an auxiliary winding and stator salient poles will give rise to a machine capable of starting asynchronously and pulling in and running at synchronous speed without DC excitation.

It seems therefore reasonable to expect that saliency on the stator of the rotary machine could also result in synchronous operation without DC excitation. In this mode, the machine would be operating as a stepping motor just like the screw-thread machine.

### CONCLUSION

The magnetics of an idealized asynchronous reluctance machine with no rotor conductors have been presented, using a two-stack machine with no rotor conductors as the starting point for the presentation. Unlike the conventional machines where torque is produced due to the interaction of two magnetic fields, one on the rotor and the other on the stator, the action of the machine is based on the reluctance torque/force between the salient pole of the rotor and a component of the overall combination of the main and auxiliary winding mmfs which moves at the speed of the rotor and consequently impart reluctance torque on the rotor. This condition obtains at any speed and adjustment is automatic. The speed of the rotor is proportional to the difference between the

frequency of the current in the main winding and that of the current in the auxiliary winding, regardless of the mode of operation, asynchronous or synchronous or DC.

The magnetics of the machine show that the machine is an asynchronous version of the screw-thread motor. If, however, the auxiliary winding is removed and salient poles are introduced on the stator instead, with the axes coinciding with that of the auxiliary winding, the machine will be capable of running as a synchronous reluctance machine like the screw thread motor. This configuration will find application in small machines with small rotors and with little space for rotor conductors. So, when two such machines are operated in cascade, the DC on the stator enables the set to be run synchronously.

From the analogy between the axial motion and the screw-thread motor, it is clear that if we have introduced stator saliency in the rotating machine, synchronous reluctance operation at

$\omega = \frac{\omega_0}{2}$  with only the primary winding excited

will be obtained. Axial motion is obtained if the conventional salient pole rotor with the usual circumferential saliency is replaced with a rotor shaft having longitudinal saliency. In this case, the auxiliary winding will have mutual coupling with the primary winding and would result in motion of the armature in the axial direction. A combination of circumferential and longitudinal saliencies in the rotor will lead to a machine having two degrees of motion in the axial and rotary directions

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